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Here we will learn about polygons, including regular polygons, angles in polygons, and complex polygons. There are also polygons worksheets based on Edexcel, AQA and OCR exam questions, along with further guidance on where to go next if you're still stuck. A regular polygon is a 2D shape where the sides are all straight line segments of equal length and each interior angle in the shape is equal. You need to be able to classify geometric shapes based on their properties, and find unknown angles and side lengths in any triangle, quadrilateral and regular polygon. E.g. Equilateral triangleSquareRegular pentagon To do this, we need to look closely at the properties of these shapes. Regular polygons have sides of equal length and angles of equal size. The table below gives the name of several regular polygon. PolygonNumber of sidesImage (regular polygon)Equilateral triangle3Square (quadrilateral)4Regular pentagon5Regular hexagon6Regular heptagon7Regular octagon8Regular nonagon 9Regular decagon10Regular hendecagon11Regular dodecagon12Regular heptadecagon17Regular icosagon20N-gonn The sum of interior angles can be calculated using the formula: Sum of interior angles = (n-2) × 180^{\circ} where n represents the number of sides. As the size of each angle is equal, we can determine the size of each angle by dividing the sum of the interior angles by the number of sides: Exterior angles are supplementary to the interior angle: Sum of exterior angles of a polygon = 360° We can use this property to find either the interior angle, or exterior angle at a vertex. As the sum of exterior angles is always 360° , for any regular polygon we can divide 360 by the number of sides to work out an exterior angle. All regular polygons can be inscribed (enclosed) in a circle. All regular polygons are known as convex polygons as all of the interior angles are less than 180° . E.g. Convex hexagonConcave hexagonAll interior angles are either acute or obtuse.At least one angle is greater than 180°. In order to classify a regular polygon: State/calculate the number of sides of the polygon. Determine the size of the angles/side lengths within the polygon. Recognise the other properties of the polygon. Get your free regular polygon worksheet of 20+ questions and answers. Includes reasoning and applied questions. DOWNLOAD FREE x Get your free regular polygon worksheet of 20+ questions and answers. Includes reasoning and applied questions. DOWNLOAD FREE Regular polygon is part of our series of lessons to support revision on polygons. You may find it helpful to start with the main polygons lesson for a summary of what to expect, or use the step by step guides below for further detail on individual topics. Other lessons in this series include: Polygons Irregular polygon Types of quadrilaterals Hexagon shape Triangles Given that ABD is an isosceles triangle, and ADE is a right angle triangle, classify the polygon BCD : State/calculate the number of sides of the polygon. BCD has 3 sides and so it is a triangle. 2Determine the size of the angles and/or side lengths within the polygon. As ABD is an isosceles triangle, BAD = BDA = (180 - 120) ÷ 2 = 30° . DAE = 180 - (90+60) = 30° as angles in a triangle total 180° . Angle ACE = 180 - (30+30+60) = 60°. Angle CBD is corresponding to angle BAE so angle CBD = 60°. This is the same for angle CDB as it is corresponding to angle DEA. 3Recognise the other properties of the polygon. All the angles in BCD are equal to 60° . The polygon is an equilateral triangle, which is a regular polygon. The polygon ABCD has two pairs of parallel sides. The diagonals are perpendicular to each other with AC = BD . Classify the quadrilateral. State/calculate the number of sides of the polygon. The polygon has four sides so we are looking at a quadrilateral. Determine the size of the angles/side lengths within the polygon. As AC = BD , the lengths of OA, OB, OC and OD are all equal as O is the centre of the polygon. Each triangle within the polygon is therefore an isosceles triangle, or equilateral. As the angle AOD = 90° (perpendicular lines meet at 90°), the two other angles in the triangle are equal to (180 - 90) ÷ 2 = 45°. Recognise the other properties of the polygon. The angle at each vertex is equal to 45 + 45 = 90°. ABCD is a square. The polygon below can be split into eight congruent triangles. One of the angles AOB = 45° . Classify the polygon. State/calculate the number of sides of the polygon. The polygon has eight sides so it is an octagon. Determine the size of the angles and/or side lengths within the polygon. As the angle at O for each isosceles triangle is equal to 45° , the other two angles must be equal to (180 - 45) ÷ 2 = 67.5°. Two adjacent angles are therefore equal to 67.5 × 2 = 135°. Recognise the other properties of the polygon. As all the interior angles are equal, the polygon is a regular octagon. Two isosceles trapezia are joined together on their longest side to form another polygon. AB = BC . Given the information in the diagram below, determine the classification of the new polygon. State/calculate the number of sides of the polygon. The new polygon has 2 joining sides so we need to reduce the total number of sides by 2 : 8 - 2 = 6. The polygon is a type of hexagon. Determine the size of the angles and/or side lengths within the polygon. As the lines AB and CD are parallel, and we know the original polygon is an isosceles trapezia, we can say that the angle at B is 120° , the angle at C and D are equal to 60° as they are co-interior angles to the angles at A and B respectively. This means that when the two tapezia are placed together the angles in the new image are a reflection of the original. The angles at C and D are double their original so all of the angles in the hexagon are equal to 120° . Recognise the other properties of the polygon. As all the interior angles are equal to 120° , and with AB = BC , all the side lengths are equal so the new polygon is a regular hexagon. A polygon has n sides. Each exterior angle is equal to 30° . All the sides are the same length. Determine the classification of the polygon. State/calculate the number of sides of the polygon. As each exterior angle is equal to 30° , we can calculate the number of sides of the polygon using the formula E_{n} = 360 ÷ n where n is the number of sides and E is the exterior angle: 30 = 360 ÷ n 30 × n = 360 n = 360 ÷ 30 n = 12 Determine the size of the angles and/or side lengths within the polygon. As we know the size of the exterior angle, we can calculate the interior angle at each vertex by using the formula I_{n} + E_{n} = 180° where I is the interior angle for the number of sides n : I_{n}+ 30 = 180 I_{n}= 150°. Each interior angle is equal to 150° . Recognise the other properties of the polygon. As all of the interior angles are equal, and the polygon has 12 sides, the polygon is a regular dodecagon. The interior angles of a polygon are: 3x + 15, 2x + 30 and 5x - 15 . Classify the polygon. State/calculate the number of sides of the polygon. As there are 3 interior angles, the polygon is a type of triangle. Determine the size of the angles and/or side lengths within the polygon. As the sum of angles in a triangle total 180° , 3x + 15 + 2x + 30 + 5x - 15 = 180 10x+30=180 10x = 150 x = 15 As x = 15 3x + 15 = 3\times15 + 15 = 60° 2x + 30 = 2\times15 + 30 = 60° 5x - 15 = 5\times15 - 15 = 60° Recognise the other properties of the polygon. As all the angels in the polygon are equal to 60° , the triangle is an equilateral triangle (a regular polygon). Make sure you know your angle properties. Getting these confused causes quite a few misconceptions. Angles in a triangle total 180°. Angles in a quadrilateral total 360°. Angles on a straight line total 180°. Incorrect quadrilateral classification There are many quadrilaterals and it is common to confuse the properties, especially for a rhombus, parallelogram, or trapezium. As well as this, stating that the polygon is a quadrilateral is not enough information for a classification. Incorrect assumptions for triangles Assuming a triangle is isosceles or equilateral can have an impact on the size of angles within the rest of the polygon, so make sure you can explain why you have chosen a specific type of triangle. As well as this, stating that the polygon is a triangle is not enough information for a classification. You must state whether it is isosceles/ equilateral etc. As the triangle has a rotational symmetry of 3 , the interior angles at A, C, and E are equal to 60^{\circ} . As the four triangles are congruent, each interior angle is equal to 60^{\circ} . The triangle BDF is equilateral. Each angle at A, B, C, and D is equal to 90^{\circ} as each diagonal passes through the centre of the circle so the angle in the semicircle is equal to 90^{\circ} . Looking at the quadrilateral AOMH , we can calculate the size of angle x by using the fact that angles in a quadrilateral total 360^{\circ} : 2x+135+90=360 2x=135 x=67.5^{\circ} Looking at the triangle, as the line MO is parallel to the line AB , angle OAB is alternate to the angle AOM and so angle OAB = x = 67.5^{\circ} . This means that triangle ABO is isosceles as the angle at A and the angle at B are both equal. Angle AOB = 45^{\circ} as angles in a triangle total 180^{\circ} . Angle BAH = 67.5 + 67.5 = 135^{\circ} . Angle ABC = 67.5 + 67.5 = 135^{\circ} and Angle BCN = 135^{\circ} as shape BCNO is congruent to shape AOMH . The line MN bisects the shape into two equal halves and so the angles on the opposite side of this line are mirrored. All of the interior angles of the octagon are equal to 135^{\circ} and so the polygon is a regular octagon. As each line segment is the same length, each triangle must be equilateral. The interior angles are all obtuse. The only way to arrange the triangles to produce a 6 sided shape with all obtuse angles is: Each interior angle is equal to 120^{\circ} as two adjacent angles of two equilateral triangles are 60 + 60 = 120^{\circ} . 360 ÷ 45 = 8 sides. All the side lengths are the same as stated in the question. 10x+8+12x-12+9x+18+6x+48+13x-22=540 50x+40=540 50x=500 x=10 As x=10 10x+8=10\times10+8=108^{\circ} 12x-12=12\times10-12=108^{\circ} 9x+18=9\times10+18=108^{\circ} 6x+48=6\times10+48=108^{\circ} 13x-22=13\times10-22=108^{\circ} All 5 of the angles are equal to 108^{\circ} . 1. (a) Two regular polygons with the same number of sides join down one side. The angle between the polygons is equal to 120^{\circ} . What type of polygons are they? (b) Calculate the number of sides of a polygon with an internal angle of 156^{\circ} . (4 marks) (a) (360 – 120)\div2 = 120^{\circ} (1) Regular hexagons (1) (b) 180 – 156 = 24^{\circ} (1) 360\div24 = 15 sides (1) 2. (a) The polygon ABCDE is made from 2 congruent isosceles triangles and two congruent right angle triangles. The point M is the midpoint of the line DE . Angle CDE and BAE = 108^{\circ} . Angle BME = 90^{\circ} and BM is a line of symmetry. Classify the following polygon ABCDE . (b) Does this shape tessellate? Explain your answer. (7 marks) (a) AED = BCD = 108^{\circ} due to symmetrical properties (1) Angles in a pentagon total 540^{\circ} or (180\div(n-2)) (1) Angle ABC = 540 – (108\times4) = 108^{\circ} (1) All sides equal as the polygon has a line of symmetry (1) Regular pentagon (1) (b) No (1) 360\div108 is not an integer (1) 3. The pentagram below is made up of 5 isosceles triangles and a regular pentagon. Calculate the interior angles for the pentagram labelled x, y, and z . Give reasons for your answer. (6 marks) x=108^{\circ} (1) Interior angle of a regular pentagon is 108^{\circ} (1) y=72^{\circ} (1) Angles on a straight line total 180^{\circ} (1) z=36^{\circ} (1) Angles in an isosceles triangle (1) You have now learned how to: Plot specified points and draw sides to complete a given polygon Distinguish between regular and irregular polygons based on reasoning about equal sides and angles Compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons Describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric Derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons Symmetry Area How to work out perimeter Prepare your KS4 students for maths GCSEs success with Third Space Learning. Weekly online one to one GCSE maths revision lessons delivered by expert maths tutors. Find out more about our GCSE maths tuition programme. We use essential and non-essential cookies to improve the experience on our website. Please read our Cookies Policy for information on how we use cookies and how to manage or change your cookie settings.AcceptPrivacy & Cookies Policy Polygons are defined as two-dimensional closed shapes that are formed by joining three or more line segments with each other. We tend to encounter polygons mostly while we learn about geometry. In this lesson, let us learn about polygons definition, regular polygons, polygon sides, and the properties of polygons, along with polygon examples and their identification. What are Polygons? The word 'Polygon' is derived from a Greek word in which 'poly' means 'many' and 'gon' means 'angle'. This means that a polygon is a closed figure that is formed by straight lines and these straight lines form the interior angles in it. Polygons can be commonly seen around us. For example, the shape of a honeycomb is a polygon with 6 sides and is known as a hexagon. Each polygon is different in structure and is categorized based on the number of sides and its properties. It should be noted that all polygons are closed plane shapes. Polygon Definition In geometry, the definition of a polygon is given as a closed two-dimensional figure which is formed by three or more straight lines. Properties of Polygons The properties of polygons help us identify them easily. In other words, the following characteristics of a polygon help us to easily check whether a given shape is a polygon or not A polygon is a closed shape, that is, there is no end that is left open in the shape. It ends and begins at the same point. It is a plane shape, that is, the shape is made of line segments or straight lines. It is a two-dimensional figure, that is, it has only two dimensions length and width. There is no depth or height to it. It has three or more sides in it. The angles in the polygon may or may not be the same. The length of the sides of a polygon may or may not be the same. Polygon Sides The sides of a polygon define the name of the specific polygon because different polygons have different number of sides. For example, if a polygon has 3 sides, then it is called a triangle, whereas, if a polygon has 4 sides, it is a quadrilateral. The following section shows the different types of polygons along with their names based on the number of sides. Types of Polygons There are different types of polygons and they have different names depending on the number of sides that they have. For example, a 3-sided polygon is a triangle, a 4-sided polygon is a quadrilateral, a 5-sided polygon is a pentagon, a 6-sided polygon is a hexagon, and so on. Polygon Chart The following chart shows the naming convention of polygons on the basis of the number of sides that they have. Each polygon is given a special name on the basis of its number of sides. For example, the trigon, also known as the triangle is made of two words 'tri' which means three, and 'gon' means angles. This shows that it is a shape that has three angles. Observe the table given below to see the names of different polygons as per their number of sides. Difference Between Regular and Irregular Polygons A polygon can be categorized as a regular or irregular polygon based on the length of its sides and the measure of its angles. The difference between a regular and irregular polygon is given in the following table. Criterion of Difference Regular Polygon Irregular Polygon Length of sides Equal Unequal Measurement of all interior angles Equal Unequal Measurement of all exterior angles Equal Unequal Regular Polygons and Irregular Polygons It is said that as per Euclidean Geometry, a polygon that is equiangular and equilateral is called a regular polygon while a polygon whose sides are not equiangular and equilateral is referred to as an irregular polygon. Regular polygons are always convex, i.e., all the interior angles measure less than 180º. In simple words, a regular polygon has all angles of the same measure at each vertex, and all sides of the same length; while a polygon that has sides of different lengths and angles of different measures is referred to as an irregular polygon. Observe the figure of a regular hexagon given below to understand the parts of a regular polygon. Here, Vertices of the hexagon: A, B, C, D, E, and F All the sides of this regular hexagon are equal, i.e., AB = BC = CD = DE = EF = FA All the interior angles are equal (represented in blue) All the exterior angles are equal (represented in yellow) BE is the diagonal. Angles in a Regular Polygon As we learned above, there are two kinds of angles that can be found in the case of a regular polygon. They are: Interior Angles of a Polygon Exterior Angles of a Polygon Interior Angles of a Polygon The interior angles are formed between the adjacent sides inside the polygon and are equal to each other in the case of a regular polygon. The number of interior angles is equal to the number of sides. The value of an interior angle of a regular polygon can be calculated if the number of sides of the regular polygon is known by using the following formula: Interior angle = 180º(n-2)/n, where 'n' is the number of sides Exterior Angles of a Polygon Each exterior angle of a regular polygon is formed by extending one side of the polygon (either clockwise or anticlockwise) and then the angle between that extension and the adjacent side is measured. Each exterior angle of a regular polygon is equal and the sum of the exterior angles of a polygon is 360°. An exterior angle can be calculated if the number of sides of a regular polygon is known by using the following formula: Exterior Angle = 360º/n, where 'n' is the number of sides of the polygon Important Notes on Polygons Polygons are 2-D figures with more than 3 sides. The angles of a regular polygon can be measured by using the following formulas: Exterior Angle = 360º/n Interior angle = 180º(n-2)/n, where n refers to the number of sides. The sum of interior and exterior angles at a point is always 180º as they form a linear pair of angles For an 'n'-sided polygon, the number of diagonals can be calculated with this formula, n(n-3)/2 Polygon Formulas There are two basic formulas for polygons listed below: Area of polygons Perimeter of polygons Let us learn about the above-listed two polygon formulas in detail. Area of Polygons The area of a polygon is defined as the measurement of space enclosed within a polygon. The area of polygons can be found by different formulas depending upon whether the polygon is a regular or an irregular polygon. For example, a triangle is a three-sided polygon which is known as a trigon. The formula for calculating the area of the trigon (triangle) is half the product of the base and height of the triangle. It is expressed in square units like, m2, cm2, ft2. Similarly, each polygon has a different formula depending on the number of sides and the type of polygon. Perimeter of Polygons The perimeter of a polygon is defined as the distance around a polygon which can be obtained by summing up the length of all given sides. The perimeter of polygon formula = Length of Side 1 + Length of Side 2 + Length of Side 3...+ Length of side N (for an N-sided polygon). It is expressed in terms of units such as meters, cm, feet, etc. Concave and Convex Polygons Concave polygons are those polygons that have at least one interior angle which is a reflex angle and it points inwards. Concave polygons have a minimum of 4 sides and a few of the diagonals in a concave polygon may lie partly or fully outside it. It is to be noted that all concave polygons are irregular because the interior angles are not equal. On the other hand, a convex polygon has no interior angle that measures more than 180°. A convex polygon can have 3 sides and no diagonal in a convex polygon lies outside it. Polygons Worksheets The polygons worksheets helps children recognize more shapes and patterns in real life. It also develops the base of understanding and establishing the necessary basic background for geometry. Download Polygons Worksheet PDFs These math worksheets should be practiced regularly and are free to download in PDF formats. ☛ Related Articles Example 1: Identify the polygons according to their number of sides. a.) A five-sided polygon b.) A seven-sided polygon c.) Six-sided polygon Solution: a.) A five-sided polygon - Pentagon b.) A seven-sided polygon - Heptagon c.) Six-sided polygon - Hexagon Example 2: State true or false: a.) A polygon is a closed shape. b.) The perimeter of a polygon is the space enclosed within a polygon. Solution: a.) True, a polygon is a closed shape. b.) False, the perimeter of a polygon is the total length of the boundary of the polygon. Example 3: Fill in the blanks: a.) In a _ polygon all angles are of the same measure. b.) An 8-sided polygon is called an _ Solution: a.) In a regular polygon all angles are of the same measure. b.) An 8-sided polygon is called an octagon. Show Answer > go to slidego to slide Have questions on basic mathematical concepts? Become a problem-solving champ using logic, not rules. Learn the why behind math with our Cuemath's certified experts. Book a Free Trial Class FAQs on Polygons Polygons are closed, two-dimensional shapes that are formed by three or more line segments. They are closed, plane figures that are bounded by straight lines. How to Identify a Polygon? A shape is a polygon if it has the following characteristics: The shape must be a closed shape, that is, it must end and begin at the same point. The shape should be made of line segments or straight lines. The shape must be a two-dimensional figure, that is, it must have only two dimensions length and width. It must have three or more sides. The angles in the polygon may or may not be the same. The length of the side of a polygon may or may not be the same. What is the Difference Between a Regular Polygon and an Irregular Polygon? A polygon in which all sides are of equal length and all angles are of equal measure is called a regular polygon. An irregular polygon is a polygon in which the sides and angles are of different lengths and measures. ☛ Also Check: How do you know if a Polygon is Regular? Any polygon is a regular polygon if it satisfies the following criteria: The length of all its sides must be equal. All interior angles must measure the same. All the exterior angles must measure the same. What is the Interior Angle of a Regular Polygon? The angle that is formed by adjacent sides inside the polygon is referred to as the interior angle. All the interior angles in a regular polygon are equal to each other. The value of an interior angle of a regular polygon can be calculated by using the following formula, interior angle = 180º(n-2)/n, where 'n' is the number of sides. Is a Circle Considered a Polygon? No, a circle is not considered to be a polygon because it is not made up of three or more straight lines or line segments. It does not fulfill the criterion which can be used to identify a polygon. It is not made up of straight lines and it does not have an interior or exterior angle. Therefore, it can be said that a circle is not a polygon. What is an 11 Sided Polygon Called? An 11-sided polygon is referred to as Hendecagon. It is derived from two Greek words 'Hendeka' which means eleven and 'gon' which means angles. Both words refer to the shape of an eleven-sided polygon. How are Polygons Named? The name of each polygon is made of two words. The first part of the word is influenced by the Greek meaning of the number of sides it possesses and it is suffixed by the word 'gon'. For example, the word 'Hexagon' is made of two words 'hex' and 'gon'. The word 'hex' means the number six and 'gon' means the angles. The words 'Triangle' and 'Quadrilateral' stand as an exception in this case as they are not given as per the nomenclature. Are Polygons Always Closed Shapes? Yes, polygons are always closed shapes as they are made of three or more straight lines which begin and start at the same point. For any shape to be a polygon it is necessary that it is a closed shape. This is also one of the most important criteria used to identify a shape as a polygon. What is the Area of Polygon Shape? The total space enclosed by a polygon in a two-dimensional plane is defined as the area of a polygon. We write the unit of area of the polygon as square units such as (meters2 or centimeters2, etc.) or USCS units (inches or feet, etc). What is the Perimeter of Polygon Shape? The perimeter of a polygon is defined as the total length of the boundary of the polygon in a two-dimensional plane. The perimeter of polygons is expressed in units like centimeters, inches, feet, and so on. Are All Triangles Polygons? Yes, all triangles are polygons as they follow all the criteria of being a polygon. In the case of any triangle, whether equilateral triangle, isosceles triangle, or scalene triangle, the following criteria are always satisfied: The shape ends and begins at the same point. It is made of line segments or straight lines. It has only two dimensions that are length and width. It has three sides. The angles may or may not be the same. The length of the side of a polygon may or may not be the same. Thus the above-given points prove that all triangles are polygons. What is a Regular Polygon? A regular polygon is one in which all the sides are of equal length and all the angles are of equal measure. For example, a square is a regular polygon. What are the Properties of a Polygon? The properties of a polygon are listed below which help us identify a figure as a polygon: A polygon is a closed shape. It is made of line segments or straight lines. A polygon is a two-dimensional shape (2D shape) that has only two dimensions - length and width. A polygon has at least three or more sides. The angles and side lengths of a polygon may be of the same length or of different lengths. How to Find the Number of Sides of a Polygon? If the value of a single exterior angle of a regular polygon is given, then the number of sides of that regular polygon can be calculated using the formula, Number of sides of the regular polygon (n) = 360 ÷ exterior angle. What are the Different Types of Polygons? There are different types of polygons that are named according to the number of sides that they have. For example, a 3-sided polygon is called a triangle, a 4-sided polygon is called a quadrilateral, a 5-sided polygon is called a pentagon, a 6-sided polygon is called a hexagon, a 7-sided polygon is called a heptagon, and so on. A detailed chart of the different types of polygons is given above on this page. What is the Difference Between Concave and Convex Polygons? The differences between concave and convex polygons are given below. Concave polygons are those polygons that have at least one interior angle which is a reflex angle and it points inwards. But a convex polygon has no interior angle that measures more than 180°. Concave polygons have a minimum of 4 sides but a convex polygon can have 3 sides. A few of the diagonals in a concave polygon may lie partly or fully outside it. But no diagonal in a convex polygon lies outside it. It is to be noted that all concave polygons are irregular because the interior angles are not equal. However, a convex polygon may not always be an irregular polygon. Q1: Heptagon is a 7-sided polygon.FalseTrueQ2: A five-sided polygon is known as a pentagon.FalseTrue Regular Polygons are closed two-dimensional planar figures made of straight lines having equal sides and equal angles. These symmetrical shapes, ranging from equilateral triangles to perfect decagons, They are important in various fields such as architecture, art, and design.The sides or edges of a polygon are formed by linking end-to-end segments of a straight line to form a closed shape. The intersection of two line segments that result in an angle is referred to as a vertex or corner. A polygon is referred to be a regular polygon if all of its sides are congruent.In this article, we will discuss Regular Polygon its types and properties along with some solved examples and questions on regular polygon. Regular Polygon DefinitionRegular polygons are closed symmetric figures made with straight lines with all sides and angles equal. This symmetry gives them a balanced and uniform appearance, making them an essential concept in geometry. Squares, rhombuses, equilateral triangles, and other shapes serve as cases of regular polygons. They have both congruent angles and congruent sides. They are equiangular shapes.Parts of PolygonA polygon has 3 parts: Angles: Angles are the geometric figures formed by joining the sides of the polygon.Interior Angle: Angles within the enclosed area of the polygonExterior Angle: Angles formed outside the polygon by extending one side.Sides: A line segment that joins two vertices is known as a side.Vertices: The point at which two sides meet is known as a vertex.Regular Polygons ExamplesThere are many examples of regular polygons in real life around us. A common example is a stop sign, an octagon with eight equal sides. Road signs often employ triangles, squares, and pentagons too. Nature showcases these shapes in beehives (hexagons) and snowflakes (hexagrams). Regular polygons' uniformity and symmetry make them prevalent in architecture, such as the facades of buildings and decorative tiles.The image added below shows regular and irregular polygons.Fun Fact : The equal sides ensure that regular polygons can perfectly fit within circumscribed and inscribed circles, further demonstrating their geometric perfection.Regular Polygon ShapesThere are various regular polygons, such as equilateral triangles, squares, regular pentagons, etc. There can be any number of regular polygons based on the number of sides they have. If it has three sides, it is an equilateral triangle. If it has four sides, it is a square. If it has five sides, it is a regular pentagon, and so on. Let's discuss these individual shapes:Equilateral TriangleProperties of Equilateral Triangle 3 Equal sides, 3 Vertices, 3 Angles.The sum of the interior angle of an equilateral triangle is 180°Each interior angle is 60° and each exterior angle is 120°.No of Diagonals : 0.No of triangle formed : 1Axis of symmetry : 3SquareProperties of Square4 Equal sides, 4 Vertices, 4 Angles.The sum of the interior angle of an equilateral triangle is 360°Each interior angle is 90° and each exterior angle is 90°.No of Diagonals : 2.No of triangle formed : 2Axis of symmetry : 4PentagonProperties of Pentagon5 Equal sides, 5 Vertices, 5 Angles.The sum of the interior angle of an equilateral triangle is 540°Each interior angle is 108° and each exterior angle is 72°.No of Diagonals : 5.No of triangle formed : 3Axis of symmetry : 5HexagonProperties of Hexagon6 Equal sides, 6 Vertices, 6 Angles.The sum of the interior angle of an equilateral triangle is 720°Each interior angle is 120° and each exterior angle is 60°.No of Diagonals : 9.No of triangle formed : 4Axis of symmetry : 6HeptagonProperties of Heptagon7 Equal sides, 7 Vertices, 7 Angles.The sum of the interior angle of an equilateral triangle is 900°Each interior angle is 128.57° and each exterior angle is 51.43°.No of Diagonals : 14.No of triangle formed : 5Axis of symmetry : 7OctagonProperties of Octagon8 Equal sides, 8 Vertices, 8 Angles.The sum of the interior angle of an equilateral triangle is 1080°Each interior angle is 135° and each exterior angle is 45°.No of Diagonals : 20.No of triangle formed : 6Axis of symmetry : 8Properties of Regular PolygonsThe general properties of the Regular Polygons are discussed below,Sum of Interior Angles of a Regular PolygonSum of Interior Angles of a Regular Polygon is given using the formula,Sum of Interior Angles = 180°(n - 2)where "n" represents the number of sides of a regular polygonEach Interior Angle of a Regular PolygonEach Interior angle of an n-sided regular polygon is measured using the formula, Each Interior Angle = [(n - 2) x 180°]/nwhere "n" represents the number of sides of a regular polygonExterior Angle of a Regular PolygonEach Exterior Angle of an n-sided regular polygon is measured using the formula,Each Exterior Angle =360°/nwhere "n" represents the number of sides of a regular polygonNumber of Diagonals in a Regular PolygonNumber of diagonals in an n-sides polygon is given using the formula,Number of Diagonals = n(n - 3)/2where "n" represents the number of sides of a regular polygonNumber of Triangles in a Regular PolygonNumber of Triangles that can be generated inside by connecting diagonals of a Regular n - sided Polygon is given using this formulaNumber of triangles = (n - 2)where "n" represents the number of sides of a regular polygonNumber of Axis of symmetry in a Regular PolygonThe number of Axis of Symmetry in an n-sides polygon (imaginary line dividing a shape into two equal halves)number of Axis of Symmetry = nwhere "n" represents the number of sides of a regular polygonRegular Polygons Formulas Regular polygons are two-dimensional closed figures with finite straight lines, as we have explained. It is made up of straight lines that join. The formulas used in a regular polygon are listed below. Area of Regular Polygon The region that the regular polygon occupies is known as its area. A polygon is classified as a triangle, quadrilateral, pentagon, etc. based on how many sides it has. The area of regular polygon is determined by: Area of Regular Polygon (A) = [l2n]/[4tan(π/n)] units2where,l is the side lengthn is the number of sidesExample: Determine the area of a polygon with 5 sides and a side length of 5 centimeters.Solution:Given,Method for determining the region is,A = [l2n]/[4tan(π/n)] A = [52 x 5] / [4 tan(180/5)]A = 125 / 4 x 0.7265A = 43.014 cm2Thus, the area of the polygon with five(5) sides is 43.014 cm2Regular Polygon Perimeter FormulaThe Perimeter of an n-sides regular polygon is can be calculated using the formula. Perimeter (P) = n × swhere,"n" represents the number of sides of a regular polygon"s" represents the length of the side of Regular PolygonExample: Find the perimeter of the hexagon with a length of 7 cm.Solution:Given,Length of Side = 7 cmFor Hexagon,n = 6Perimeter of Regular Polygon(P) = n × sP = 6 × 7 = 42 cmThus, the perimeter of the hexagon is 42 cmIrregular Polygon Vs Regular PolygonRegular polygons provide symmetrical forms and consistency which makes them simple to recognize. Contrarily, irregular polygons are less predictable and more difficult to categorize since their sides and angles have various lengths and measurements, resulting in asymmetrical forms. The major differences between a Regular polygon and an Irregular polygon are discussed in the table below,Irregular PolygonRegular PolygonLengths of the sides vary polygonEach side is of the same length for the polygon.Interior angles in the polygon are different.Each internal angles in the polygon are the same.Examples include irregular forms that are not consistent.Squares, Triangles, Pentagons, Hexagons, and other shapes with equal sides are examples of regular polygons.Identification and classification might be difficult due to their diverse features.Simple to classify and identify.Read More,Polygon FormulaTypes of PolygonsArea of Regular PolygonSolved Examples on Regular PolygonsExample 1: If a polygon has 40 external angles, then the number of sides it has,Solution:GivenEach Exterior Angle(n) = 40° Number of Sides = 360°/n [Formula of regular polygon on the Exterior Angles]Number of Sides = 360°/40° = 9Thus, the number of sides in the polygon is 9Example 2: Calculate the number of diagonals in a regular polygon with 24 sides.Solution:Given,Number of sides in regular polygon = 24 sidesFormula for diagonals of the regular polygonNumber of diagonals in n sides polygon(N) = n(n - 3)/2N = 24(24-3)/2 = 252Thus, the number of diagonals in a polygon with 24 sides is 252Example 3: What is the number of sides of a regular polygon if each interior angle is 90°?Solution:Given,Each Interior Angle = 90°Formula of interior angle of an n-sided regular polygon,Each Interior Angle = [(n - 2) x 180°]/n90° = [(n - 2) x 180°]/n90n = (n - 2) x 180°90n = 180n - 36090n-180n = - 360-90n = -360n = 4Thus, the number of sides in the regular polygon with 90 degree interior angle is 4Conclusion Regular polygons, with their equal sides and angles, are a fundamental concept of geometric. Their symmetry and uniformity not only make them unique but also incredibly useful in various applications, from architecture to everyday design. Their predictable and uniform structure makes them ideal for creating patterns, tiling surfaces, and designing various objects that require both form and function. A polygon is a plane shape (two-dimensional) with straight sides. Polygons are all around us, from doors and windows to stop signs. Let's explore what makes some of them 'regular' and why that matters! Regular A regular polygon has: all sides equal and all angles equal. Otherwise it is irregular. Regular Pentagon Irregular Pentagon Regular polygons have some cool properties, let's explore them. Properties of Regular Polygons So what can we know about regular polygons? First of all, we can work out angles. The Exterior Angle is the angle between any side of a shape, and a line extended from the next side. images/exterior-angles.js All the exterior angles of a polygon add up to 360°, so: Each exterior angle must be 360°/n (where n is the number of sides) Press play button to see. Imagine walking around the edge of a building, turning at each corner. By the time you return to where you started, you've turned a full 360° Exterior Angle (of a regular octagon) An octagon has 8 sides, so: Exterior angle =360° / n =360° / 8 =45° Interior Angles The Interior Angle and Exterior Angle are measured from the same line, so they add up to 180°. Interior Angle = 180° − Exterior Angle Since we already know the Exterior Angle is 360°/n: Interior Angle = 180° − 360°/n Which can be rearranged like this: Interior Angle= 180° − 360°/n = (n × 180° / n) − (2 × 180° / n) = (n−2) × 180°/n So we also have this: Interior Angle = (n−2) × 180° / n A regular octagon has 8 sides, so: Exterior Angle = 360° / 8 = 45° Interior Angle = 180° − 45° = 135° Interior Angle (of a regular octagon) Or we could use: Interior Angle= (n−2) × 180° / n = (8−2) × 180° / 8 = 6 × 180° / 8 = 135° Same answer. A regular hexagon has 6 sides, so: Exterior Angle = 360° / 6 = 60° Interior Angle = 180° − 60° = 120° Some polygons have special circles hidden inside and around them! Let's explore them and their names: "Circumcircle, Incircle, Radius and Apothem ..." Sounds quite musical if you repeat it a few times, but they are just the names of the "outer" and "inner" circles (and each radius) that can be drawn on a polygon like this: The "outside" circle is called a circumcircle, and it connects all vertices (corner points) of the polygon. The radius of the circumcircle is also the radius of the polygon. The "inside" circle is called an incircle and it just touches each side of the polygon at its midpoint. The radius of the incircle is the apothem of the polygon. (Not all polygons have those properties, but all triangles and regular polygons do). Breaking into Triangles We can learn a lot about regular polygons by breaking them into triangles like this: Notice that: the "base" of the triangle is one side of the polygon. the "height" of the triangle is the "Apothem" of the polygon Now, the area of a triangle is half of the base times height, so: Area of one triangle = base × height / 2 = side × apothem / 2 To get the area of the whole polygon, just add up the areas of all the little triangles ("n" of them): Area of Polygon = n × side × apothem / 2 And since the perimeter is all the sides = n × side, we get: Area of Polygon = perimeter × apothem / 2 A Smaller Triangle By cutting the triangle in half we get this: (Note: The angles are in radians, not degrees) The small triangle is right-angled and so we can use sine, cosine and tangent to find how the side, radius, apothem and n (number of sides) are related: sin(π/n) = (Side/2) / Radius Side = 2 × Radius × sin(π/n) cos(π/n) = Apothem / Radius Apothem = Radius × cos(π/n) tan(π/n) = (Side/2) / Apothem Side = 2 × Apothem × tan(π/n) There are a lot more relationships like those (most of them just "re-arrangements"), but these are the most useful ones. More Area Formulas We can use that to calculate the area when we only know the Apothem: Area of Small Triangle= ½ × Apothem × (Side/2) And we know (from the "tan" formula above) that: Side = 2 × Apothem × tan(π/n) So: Area of Small Triangle= ½ × Apothem × (Apothem × tan(π/n)) = ½ × Apothem2 × tan(π/n) And there are 2 such triangles per side, or 2n for the whole polygon: Area of Polygon = n × Apothem2 × tan(π/n) When we don't know the Apothem, we can use the same formula but re-worked for Radius or for Side: Area of Polygon = ½ × n × Radius2 × sin(2 × π/n) Area of Polygon = ¼ × n × Side2 / tan(π/n) A Table of Values And here is a table of Side, Apothem and Area compared to a Radius of "1", using the formulas we have worked out: Type Name when Regular Sides (n) Shape Interior Angle Radius Side Apothem Area Triangle (or Trigon) Equilateral Triangle 3 60° 1 1.732 (√3) 0.5 1.299 (¾√3) Quadrilateral (or Tetragon) Square 4 90° 1 1.414 (√2) 0.707 (1/√2) 2 Pentagon Regular Pentagon 5 108° 1 1.176 0.809 2.378 Hexagon Regular Hexagon 6 120° 1 1 0.866 (½√3) 2.598 ((3/2)√3) Heptagon (or Septagon) Regular Heptagon 7 128.571° 1 0.868 0.901 2.736 Octagon Regular Octagon 8 135° 1 0.765 0.924 2.828 (2√2) ... ... Pentacontagon Regular Pentacontagon 50 172.8° 1 0.126 0.998 3.133 (Note: values correct to 3 decimal places only) Graph And here is a graph of the table above, but with number of sides ("n") from 3 to 30. Notice that as "n" gets bigger, the Apothem is tending towards 1 (equal to the Radius) and that the Area is tending towards π = 3.14159..., just like a circle. What is the Side length tending towards? 6039, 6042, 1794, 1792, 1793, 1795, 1796, 1797, 1798, 1799 Copyright © 2025 Rod Pierce A polygon is a closed geometric shape made of a finite number of straight line-segments joined end to end. In Greek, 'poly' means 'many,' and 'gon' means 'angle.' It should have a minimum of three sides but can have any maximum number of sides. A polygon is usually named based on the number of sides it contains. A polygon with n sides is called an n-gon. Given below is the list of common polygons: Polygon Shape As shown in the above image, the most basic types of polygons found in everyday life are: 1) triangle, 2) quadrilateral, 3) pentagon, 4) hexagon, 5) heptagon, 6) octagon, 7) nonagon, and 8) decagon. Given below is the list of the names of polygons with their basic properties: Types of Polygon a) Convex Polygon It is a polygon that has all its interior angles less than 180°. All the diagonals of a convex polygon lie inside the closed figure. Triangles, all convex quadrilaterals, regular pentagon, and regular hexagon are common examples of a convex polygon. b) Concave Polygon Also known as a non-convex polygon, it is a polygon having at least one of its interior angles measuring more than 180°. Some diagonals of a concave polygon lie outside the closed figure. A dart or an arrowhead in quadrilaterals, some irregular pentagon and hexagon are common examples of the concave polygon. a) Regular Polygon It is a polygon having all sides of equal length and all interior angles of equal measure. A regular polygon is thus both equilateral and equiangular. Triangle, quadrilateral, pentagon, and hexagon are common examples of the regular polygon. b) Irregular Polygon It is a polygon that has sides of unequal length and angles of unequal measure. Any polygon that is not regular is thus an irregular polygon. Scalene triangle, quadrilaterals such as a rectangle, trapezoid or a kite, irregular pentagon, and hexagons are common examples of the irregular polygon. Pentagon, the large five-sided building that serves as the headquarters of the U.S. Department of DefenseThe Pyramids of Egypt with five sidesThe Bermuda TriangleThe rectangle-shaped screen of our laptop, a television set, and mobile phonesThe tiles on which we walk in our homes and streetsThe wall of a building, the truss of a bridge Q1. Are all quadrilaterals polygons? Ans. Yes, all quadrilaterals are polygons. Since a quadrilateral has four sides, it is a four-sided polygon. Q2. Are all triangles polygons? Ans. Yes, all triangles, by definition, are polygons with three sides. Q3. Are circles polygons? Ans. No, circles are not polygons because they have no sides. Q4. Is a cube a polygon? Ans. No, a cube is not a polygon. Polygons are flat, two-dimensional, closed shapes with sides made of straight lines, whereas a cube is a three-dimensional solid figure. Q5. Is a star a polygon? Ans. Yes, a star is a polygon. In geometry, a star is a special type of non-convex polygon that is commonly also known as a star polygon. Q6. What are the names of polygons with right angles? Ans. Square, rectangle, and right trapezoid are three polygons that have right angles. Q7. Do all polygons have parallel sides? Ans. Every regular polygon with an even number of sides has pairs of parallel sides. Regular polygons have half as many parallel sides as the total number of sides. Q8. Is a heart a polygon? Ans. No, a heart is not a polygon because a heart is made of curved sides. A polygon, on the other hand, is made of straight-line segments. Q9. Can a polygon have curved sides? Ans. No, the sides of a polygon are made of straight-line segments. Thus any geometric shape that has curved sides is not a polygon. Q10. Is an oval a polygon? Ans. An oval is not a polygon because it has no straight sides. Q11. Can a polygon have more sides, or more angles? Ans. No, polygons have the same number of sides and angles because they are closed figures with non-intersecting lines. Last modified on June 8th, 2024 Equiangular and equilateral polygon Regular polygonEdges and vertices n {\displaystyle n} Schläfli symbol { n } {\displaystyle \{n\}} Coxeter–Dynkin diagramSymmetry groupDn, order 2nDual polygonSelf-dualArea (with side length s {\displaystyle s}) A = 1 4 n s 2 cot (π n) {\displaystyle A={\tfrac {1}{4}}ns^{2}\cot \left({\frac {\pi }{n}}\right)} Internal angle (n − 2) × π n {\displaystyle (n-2)\times {\frac {\pi }{n}}} Internal angle sum (n − 2) × π {\displaystyle \left(n-2\right)\times {\pi }} Inscribed circle diameter d IC = s cot (π n) {\displaystyle d_{\text{IC}}=s\cot \left({\frac {\pi }{n}}\right)} Circumscribed circle diameter d OC = s csc (π n) {\displaystyle d_{\text{OC}}=s\csc \left({\frac {\pi }{n}}\right)} PropertiesConvex, cyclic, equilateral, isogonal, isotoxal In Euclidean geometry, a regular polygon is a polygon that is direct equiangular (all angles are equal in measure) and equilateral (all sides have the same length). Regular polygons may be either convex or star. In the limit, a sequence of regular polygons with an increasing number of sides approximates a circle, if the perimeter or area is fixed, or a regular apeirogon (effectively a straight line), if the edge length is fixed. Regular convex and star polygons with 3 to 12 vertices labelled with their Schläfli symbols These properties apply to all regular polygons, whether convex or star: A regular n-sided polygon has rotational symmetry of order n. All vertices of a regular polygon lie on a common circle (the circumscribed circle); i.e., they are concyclic points. That is, a regular polygon is a cyclic polygon. Together with the property of equal-length sides, this implies that every regular polygon also has an inscribed circle or incircle that is tangent to every side at the midpoint. Thus a regular polygon is a tangential polygon. A regular n-sided polygon can be constructed with compass and straightedge if and only if the odd prime factors of n are distinct Fermat primes. (See constructible polygon.) A regular n-sided polygon can be constructed with origami if and only if n = 2 a 3 b p 1 ⋯ p r {\displaystyle n=2^{a}3^{b}p_{1}\cdots p_{r}} for some r ∈ N {\displaystyle r\in \mathbb {N} } , where each distinct p i {\displaystyle p_{i}} is a Pierpont prime.[1] The symmetry group of an n-sided regular polygon is the dihedral group Dn (of order 2n): D2, D3, D4, ... It consists of the rotations in Cn, together with reflection symmetry in n axes that pass through the center. If n is even then half of these axes pass through two opposite vertices, and the other half through the midpoint of opposite sides. If n is odd then all axes pass through a vertex and the midpoint of the opposite side. All regular simple polygons (a simple polygon is one that does not intersect itself anywhere) are convex. Those having the same number of sides are also similar. An n-sided convex regular polygon is denoted by its Schläfli symbol { n } {\displaystyle \{n\}} . For n < 3 {\displaystyle n 2 {\displaystyle n>2} , the number of diagonals is 1 2 n (n − 3) {\displaystyle {\tfrac {1}{2}}n(n-3)} ; i.e., 0, 2, 5, 9, ..., for a triangle, square, pentagon, hexagon, ... The diagonals divide the polygon into 1, 4, 11, 24, ... pieces.[a] For a regular n-gon inscribed in a circle of radius 1 {\displaystyle 1} , the product of the distances from a given vertex to all other vertices (including adjacent vertices and vertices connected by a diagonal) equals n. For a regular simple n-gon with circumradius R and distances di from an arbitrary point in the plane to the vertices, we have[2] 1 n ∑ i = 1 n d i 4 + 3 R 4 = (1 n ∑ i = 1 n d i 2 + R 2) 2 . {\displaystyle {\frac {1}{n}}\sum _{i=1}^{n}d_{i}^{4}+3R^{4}={\biggl (}{\frac {1}{n}}\sum _{i=1}^{n}d_{i}^{2}+R^{2}{\biggr)}^{2}.} For higher powers of distances d i {\displaystyle d_{i}} from an arbitrary point in the plane to the vertices of a regular n-gon, if S n (2 m) = 1 n ∑ i = 1 n d i 2 m {\displaystyle S_{n}^{(2m)}={\frac {1}{n}}\sum _{i=1}^{n}d_{i}^{2m}} , then[3] S n (2 m) = (S n (2)) m + ∑ k = 1 ⌊ m / 2 ⌋ (m 2 k) (2 k k) R 2 k (S n (2) − R 2) k (S n (2)) m − 2 k {\displaystyle S_{n}^{(2m)}=\left(S_{n}^{(2)}\right)^{m}+\sum _{k=1}^{\left\lfloor m/2\right\rfloor }{\binom {m}{2k}}{\binom {2k}{k}}R^{2k}\left(S_{n}^{(2)}-R^{2}\right)^{k}\left(S_{n}^{(2)}\right)^{m-2k}} , and S n (2 m) = (S n (2)) m + ∑ k = 1 ⌊ m / 2 ⌋ 1 2 k (m 2 k) (2 k k) (S n (4) − (S n (2)) 2) k (S n (2)) m − 2 k {\displaystyle S_{n}^{(2m)}=\left(S_{n}^{(2)}\right)^{m}+\sum _{k=1}^{\left\lfloor m/2\right\rfloor }{\frac {1}{2^{k}}}{\binom {m}{2k}}{\binom {2k}{k}}\left(S_{n}^{(4)}-\left(S_{n}^{(2)}\right)^{2}\right)^{k}\left(S_{n}^{(2)}\right)^{m-2k}} , where m is a positive integer less than n. If L is the distance from an arbitrary point in the plane to the centroid of a regular n-gon with circumradius R, then[3] ∑ i = 1 n d i 2 m = n ((R 2 + L 2) m + ∑ k = 1 ⌊ m / 2 ⌋ (m 2 k) (2 k k) R 2 k L 2 k (R 2 + L 2) m − 2 k) {\displaystyle \sum _{i=1}^{n}d_{i}^{2m}=n\left(\left(R^{2}+L^{2}\right)^{m}+\sum _{k=1}^{\left\lfloor m/2\right\rfloor }{\binom {m}{2k}}{\binom {2k}{k}}R^{2k}L^{2k}\left(R^{2}+L^{2}\right)^{m-2k}\right)} , where m = 1 , 2 , … , n − 1 {\displaystyle m=1,2,\dots ,n-1} . For a regular n-gon, the sum of the perpendicular distances from any interior point to the n sides is n times the apothem[4]: p. 72 (the apothem being the distance from the center to any side). This is a generalization of Viviani's theorem for the n = 3 case.[5][6] Regular pentagon (n = 5) with side s, circumradius R and apothem a Graphs of side, s; apothem, a; and area, A of regular polygons of n sides and circumradius 1, with the base, b of a rectangle with the same area. The green line shows the case n = 6. The circumradius R from the center of a regular polygon to one of the vertices is related to the side length s or to the apothem a by R = s 2 sin (π n) = a cos (π n) , a = s 2 tan (π n) {\displaystyle R={\frac {s}{2\sin \left({\frac {\pi }{n}}\right)}}={\frac {a}{\cos \left({\frac {\pi }{n}}\right)}}\quad ,\quad a={\frac {s}{2\tan \left({\frac {\pi }{n}}\right)}}} For constructible polygons, algebraic expressions for these relationships exist (see Bicentric polygon § Regular polygons). The sum of the perpendiculars from a regular n-gon's vertices to any line tangent to the circumcircle equals n times the circumradius.[4]:p. 73 The sum of the squared distances from the vertices of a regular n-gon to any point on its circumcircle equals 2nR2 where R is the circumradius.[4]:p. 73 The sum of the squared distances from the midpoints of the sides of a regular n-gon to any point on the circumcircle is 2nR2 − 1/4ns2, where s is the side length and R is the circumradius.[4]:p. 73 If d i {\displaystyle d_{i}} are the distances from the vertices of a regular n {\displaystyle n} -gon to any point on its circumcircle, then [3] 3 (∑ i = 1 n d i 2) 2 = 2 n ∑ i = 1 n d i 4 {\displaystyle 3{\biggl (}\sum _{i=1}^{n}d_{i}^{2}{\biggr)}^{2}=2n\sum _{i=1}^{n}d_{i}^{4}} . Coxeter states that every zonogon (a 2m-gon whose opposite sides are parallel and of equal length) can be dissected into (m 2) {\displaystyle {\tbinom {m}{2}}} or 1/2m(m − 1) parallelograms. These tilings are contained as subsets of vertices, edges and faces in orthogonal projections m-cubes.[7] In particular, this is true for any regular polygon with an even number of sides, in which case the parallelograms are all rhombi. The list OEIS: A006245 gives the number of solutions for smaller polygons. Examples of dissections for selected even-sided regular polygons Sides 6 8 10 12 14 16 Rhombs 3 6 10 15 21 28 Sides 18 20 24 30 40 50 Rhombs 36 45 66 105 190 300 The area A of a convex regular n-sided polygon having side s, circumradius R, apothem a, and perimeter p is given by[8][9] A = 1 2 n s a = 1 2 p a = 1 4 n s 2 cot (π n) = n a 2 tan (π n) = 1 2 n R 2 sin (2 π n) {\displaystyle {\begin{aligned}A&={\tfrac {1}{2}}nsa\\&={\tfrac {1}{2}}pa\\&={\tfrac {1}{4}}ns^{2}\cot \left({\tfrac {\pi }{n}}\right)\\&=na^{2}\tan \left({\tfrac {\pi }{n}}\right)\\&={\tfrac {1}{2}}nR^{2}\sin \left({\tfrac {2\pi }{n}}\right)\end{aligned}}} For regular polygons with side s = 1, circumradius R = 1, or apothem a = 1, this produces the following table:[b] (Since cot x → 1 / x {\displaystyle \scriptstyle \cot x\rightarrow 1/x} as x → 0 {\displaystyle \scriptstyle x\rightarrow 0} , the area when s = 1 {\displaystyle \scriptstyle s=1} tends to n 2 / 4 π {\displaystyle \scriptstyle n^{2}/4\pi } as n {\displaystyle \scriptstyle n} grows large.) Numberof sides Area when side s = 1 Area when circumradius R = 1 Area when apothem a = 1 Exact Approximation Exact Approximation Relative to circumcircle area Exact Approximation Relative to in-circle area n n 4 cot (π n) {\displaystyle \scriptstyle {\tfrac {n}{4}}\cot \left({\tfrac {\pi }{n}}\right)} n 2 sin (2 π n) {\displaystyle \scriptstyle {\tfrac {n}{2}}\sin \left({\tfrac {2\pi }{n}}\right)} n 2 π sin (2 π n) {\displaystyle \scriptstyle {\tfrac {n}{2\pi }}\sin \left({\tfrac {2\pi }{n}}\right)} n tan (π n) {\displaystyle \scriptstyle n\tan \left({\tfrac {\pi }{n}}\right)} n π tan (π n) {\displaystyle \scriptstyle {\tfrac {n}{\pi }}\tan \left({\tfrac {\pi }{n}}\right)} 3 3 4 {\displaystyle \scriptstyle {\tfrac {\sqrt {3}}{4}}} 0.433012702 3 3 4 {\displaystyle \scriptstyle {\tfrac {3{\sqrt {3}}}{4}}} 1.299038105 0.4134966714 3 3 {\displaystyle \scriptstyle 3{\sqrt {3}}} 5.196152424 1.653986686 4 1 1.000000000 2 2.000000000 0.6366197722 4 4.000000000 1.273239544 5 1 4 25 + 10 5 {\displaystyle \scriptstyle {\tfrac {1}{4}}{\sqrt {25+10{\sqrt {5}}}}} 1.720477401 5 4 1 2 (5 + 5) {\displaystyle \scriptstyle {\tfrac {5}{4}}{\sqrt {{\tfrac {1}{2}}\left(5+{\sqrt {5}}\right)}}} 2.377641291 0.7568267288 5 5 − 2 5 {\displaystyle \scriptstyle 5{\sqrt {5-2{\sqrt {5}}}}} 3.632712640 1.156328347 6 3 3 2 {\displaystyle \scriptstyle {\tfrac {3{\sqrt {3}}}{2}}} 2.598076211 3 3 2 {\displaystyle \scriptstyle {\tfrac {3{\sqrt {3}}}{2}}} 2.598076211 0.8269933428 2 3 {\displaystyle \scriptstyle 2{\sqrt {3}}} 3.464101616 1.102657791 7 3.633912444 2.736410189 0.8710264157 3.371022333 1.073029735 8 2 + 2 2 {\displaystyle \scriptstyle 2+2{\sqrt {2}}} 4.828427125 2 2 {\displaystyle \scriptstyle 2{\sqrt {2}}} 2.828427125 0.9003163160 8 (2 − 1) {\displaystyle \scriptstyle 8\left({\sqrt {2}}-1\right)} 3.313708500 1.054786175 9 6.181824194 2.892544244 0.9207254290 3.275732109 1.042697914 10 5 2 5 + 2 5 {\displaystyle \scriptstyle {\tfrac {5}{2}}{\sqrt {5+2{\sqrt {5}}}}} 7.694208843 5 2 1 2 (5 − 5) {\displaystyle \scriptstyle {\tfrac {5}{2}}{\sqrt {{\tfrac {1}{2}}\left(5-{\sqrt {5}}\right)}}} 2.938926262 0.9354892840 2 25 − 10 5 {\displaystyle \scriptstyle 2{\sqrt {25-10{\sqrt {5}}}}} 3.249196963 1.034251515 11 9.365639907 2.973524496 0.9465022440 3.229891423 1.028106371 12 6 + 3 3 {\displaystyle \scriptstyle 6+3{\sqrt {3}}} 11.19615242 3 3.000000000 0.9549296586 12 (2 − 3) {\displaystyle \scriptstyle 12\left(2-{\sqrt {3}}\right)} 3.215390309 1.023490523 13 13.18576833 3.020700617 0.9615188694 3.204212220 1.019932427 14 15.33450194 3.037186175 0.9667663859 3.195408642 1.017130161 15 15 8 (15 + 3 + 2 (5 + 5)) {\displaystyle \scriptstyle {\tfrac {15}{8}}\left({\sqrt {15}}+{\sqrt {3}}+{\sqrt {2\left(5+{\sqrt {5}}\right)}}\right)} 17.64236291 15 16 (15 + 3 − 10 − 2 5) {\displaystyle \scriptstyle {\tfrac {15}{16}}\left({\sqrt {15}}+{\sqrt {3}}-{\sqrt {10-2{\sqrt {5}}}}\right)} 3.050524822 0.9710122088 15 2 (3 3 − 15 − 2 (25 − 11 5)) {\displaystyle \scriptstyle {\tfrac {15}{2}}\left(3{\sqrt {3}}-{\sqrt {15}}-{\sqrt {2\left(25-11{\sqrt {5}}\right)}}\right)} 3.188348426 1.014882824 16 4 (1 + 2 + 2 (2 + 2)) {\displaystyle \scriptstyle 4\left(1+{\sqrt {2}}+{\sqrt {2\left(2+{\sqrt

{2}}\right)}\right)} 20.10935797 4 2 − 2 {\displaystyle \scriptstyle 4{\sqrt {2-{\sqrt {2}}}}} 3.061467460 0.9744953584 16 (1 + 2) (2 (2 − 2) − 1) {\displaystyle \scriptstyle 16\left(1+{\sqrt {2}}\right)\left({\sqrt {2\left(2-{\sqrt {2}}\right)}}-1\right)} 3.182597878 1.013052368 17 22.73549190 3.070554163 0.9773877456 3.177850752 1.011541311 18 25.52076819 3.078181290 0.9798155361 3.173885653 1.010279181 19 28.46518943 3.084644958 0.9818729854 3.170539238 1.009213984 20 5 (1 + 5 + 5 + 2 5) {\displaystyle \scriptstyle 5\left(1+{\sqrt {5}}+{\sqrt {5+2{\sqrt {5}}}}\right)} 31.56875757 5 2 (5 − 1) {\displaystyle \scriptstyle {\tfrac {5}{2}}\left({\sqrt {5}}-1\right)} 3.090169944 0.9836316430 20 (1 + 5 − 5 + 2 5) {\displaystyle \scriptstyle 20\left(1+{\sqrt {5}}-{\sqrt {5+2{\sqrt {5}}}}\right)} 3.167688806 1.008306663 100 795.5128988 3.139525977 0.9993421565 3.142626605 1.000329117 1000 79577.20975 3.141571983 0.9999934200 3.141602989 1.000003290 104 7957746.893 3.141592448 0.9999999345 3.141592757 1.000000033 106 79577471545 3.141592654 1.000000000 3.141592654 1.000000000 Comparison of sizes of regular polygons with the same edge length, from three to sixty sides. The size increases without bound as the number of sides approaches infinity. Of all n-gons with a given perimeter, the one with the largest area is regular.[10] Main article: Constructible polygon Some regular polygons are easy to construct with compass and straightedge; other regular polygons are not constructible at all. The ancient Greek mathematicians knew how to construct a regular polygon with 3, 4, or 5 sides,[11]:p. xi and they knew how to construct a regular polygon with double the number of sides of a given regular polygon.[11]:pp. 49–50 This led to the question being posed: is it possible to construct all regular n-gons with compass and straightedge? If not, which n-gons are constructible and which are not? Carl Friedrich Gauss proved the constructibility of the regular 17-gon in 1796. Five years later, he developed the theory of Gaussian periods in his Disquisitiones Arithmeticae. This theory allowed him to formulate a sufficient condition for the constructibility of regular polygons: A regular n-gon can be constructed with compass and straightedge if n is the product of a power of 2 and any number of distinct Fermat primes (including none). (A Fermat prime is a prime number of the form 2 (2 n) + 1. {\displaystyle 2^{\left(2^{n}\right)}+1.}) Gauss stated without proof that this condition was also necessary, but never published his proof. A full proof of necessity was given by Pierre Wantzel in 1837. The result is known as the Gauss–Wantzel theorem. Equivalently, a regular n-gon is constructible if and only if the cosine of its common angle is a constructible number—that is, can be written in terms of the four basic arithmetic operations and the extraction of square roots. The cube contains a skew regular hexagon, seen as 6 red edges zig-zagging between two planes perpendicular to the cube's diagonal axis. The zig-zagging side edges of a n-antiprism represent a regular skew 2n-gon, as shown in this 17-gonal antiprism. A regular skew polygon in 3-space can be seen as nonplanar paths zig-zagging between two parallel planes, defined as the side-edges of a uniform antiprism. All edges and internal angles are equal. The Platonic solids (the tetrahedron, cube, octahedron, dodecahedron, and icosahedron) have Petrie polygons, seen in red here, with sides 4, 6, 6, 10, and 10 respectively. More generally regular skew polygons can be defined in n-space. Examples include the Petrie polygons, polygonal paths of edges that divide a regular polytope into two halves, and seen as a regular polygon in orthogonal projection. In the infinite limit regular skew polygons become skew apeirogons. Regular star polygons2 < 2q < p, gcd(p, q) = 1{5/2}{7/2}{7/3}Schläfli symbol{p/q}Vertices and EdgespDensityqCoxeter diagramSymmetry groupDihedral (Dp)Dual polygonSelf-dualInternal angle(degrees) 180 − 360 q p {\displaystyle 180-{\frac {360q}{p}}} [12] A non-convex regular polygon is a regular star polygon. The most common example is the pentagram, which has the same vertices as a pentagon, but connects alternating vertices. For an n-sided star polygon, the Schläfli symbol is modified to indicate the density or "starriness" m of the polygon, as {n/m}. If m is 2, for example, then every second point is joined. If m is 3, then every third point is joined. The boundary of the polygon winds around the center m times. The (non-degenerate) regular stars of up to 12 sides are: Pentagram – {5/2} Heptagram – {7/2} and {7/3} Octagram – {8/3} Enneagram – {9/2} and {9/4} Decagram – {10/3} Hendecagram – {11/2}, {11/3}, {11/4} and {11/5} Dodecagram – {12/5} m and n must be coprime, or the figure will degenerate. The degenerate regular stars of up to 12 sides are: Tetragon – {4/2} Hexagons – {6/2}, {6/3} Octagons – {8/2}, {8/4} Enneagon – {9/3} Decagons – {10/2}, {10/4}, and {10/5} Dodecagons – {12/2}, {12/3}, {12/4}, and {12/6} Two interpretations of {6/2} Grünbaum{6/2} or 2{3}[13] Coxeter2{3} or {6}[2{3}]{6} Doubly-wound hexagon Hexagram as a compoundof two triangles Depending on the precise derivation of the Schläfli symbol, opinions differ as to the nature of the degenerate figure. For example, {6/2} may be treated in either of two ways: For much of the 20th century (see for example Coxeter (1948)), we have commonly taken the /2 to indicate joining each vertex of a convex {6} to its near neighbors two steps away, to obtain the regular compound of two triangles, or hexagram. Coxeter clarifies this regular compound with a notation {kp}[k{p}]{kp} for the compound {p/k}, so the hexagram is represented as {6}[2{3}]{6}.[14] More compactly Coxeter also writes 2{n/2}, like 2{3} for a hexagram as compound as alternations of regular even-sided polygons, with italics on the leading factor to differentiate it from the coinciding interpretation.[15] Many modern geometers, such as Grünbaum (2003),[13] regard this as incorrect. They take the /2 to indicate moving two places around the {6} at each step, obtaining a "double-wound" triangle that has two vertices superimposed at each corner point and two edges along each line segment. Not only does this fit in better with modern theories of abstract polytopes, but it also more closely copies the way in which Poinsot (1809) created his star polygons – by taking a single length of wire and bending it at successive points through the same angle until the figure closed. This section needs expansion. You can help by adding to it. (December 2024) See also: Self-dual polyhedra All regular polygons are self-dual to congruency, and for odd n they are self-dual to identity. In addition, the regular star figures (compounds), being composed of regular polygons, are also self-dual. A uniform polyhedron has regular polygons as faces, such that for every two vertices there is an isometry mapping one into the other (just as there is for a regular polygon). A quasiregular polyhedron is a uniform polyhedron which has just two kinds of face alternating around each vertex. A regular polyhedron is a uniform polyhedron which has just one kind of face. The remaining (non-uniform) convex polyhedra with regular faces are known as the Johnson solids. A polyhedron having regular triangles as faces is called a deltahedron. Euclidean tilings by convex regular polygons Platonic solid List of regular polytopes and compounds Equilateral polygon Carlyle circle ^ OEIS: A007678 ^ Results for R = 1 and a = 1 obtained with Maple, using function definition: f := proc (n) options operator, arrow; [[convert(1/4*n*cot(Pi/n), radical), convert(1/4*n*cot(Pi/n), float)], [convert(1/2*n*sin(2*Pi/n), radical), convert(1/2*n*sin(2*Pi/n), float), convert(1/2*n*sin(2*Pi/n)/Pi, float)], [convert(n*tan(Pi/n), radical), convert(n*tan(Pi/n), float), convert(n*tan(Pi/n)/Pi, float)]] end procThe expressions for n = 16 are obtained by twice applying the tangent half-angle formula to tan(π/4) ^ Hwa, Young Lee (2017). Origami-Constructible Numbers (PDF) (MA thesis). University of Georgia. pp. 55–59. ^ Park, Poo-Sung. "Regular polytope distances", Forum Geometricorum 16, 2016, 227-232. ^ a b c Meskhishvili, Mamuka (2020). "Cyclic Averages of Regular Polygons and Platonic Solids". Communications in Mathematics and Applications. 11: 335–355. arXiv:2010.12340. doi:10.26713/cma.v11i3.1420 (inactive 1 July 2025).{{cite journal}}: CS1 maint: DOI inactive as of July 2025 (link) ^ a b c d Johnson, Roger A., Advanced Euclidean Geometry, Dover Publ., 2007 (orig. 1929). ^ Pickover, Clifford A, The Math Book, Sterling, 2009: p. 150 ^ Chen, Zhibo, and Liang, Tian. "The converse of Viviani's theorem", The College Mathematics Journal 37(5), 2006, pp. 390–391. ^ Coxeter, Mathematical recreations and Essays, Thirteenth edition, p.141 ^ "Math Open Reference". Retrieved 4 Feb 2014. ^ "Mathwords". ^ Chakerian, G.D. "A Distorted View of Geometry." Ch. 7 in Mathematical Plums (R. Honsberger, editor). Washington, DC: Mathematical Association of America, 1979: 147. ^ a b Bold, Benjamin. Famous Problems of Geometry and How to Solve Them, Dover Publications, 1982 (orig. 1969). ^ Kappraff, Jay (2002). Beyond measure: a guided tour through nature, myth, and number. World Scientific. p. 258. ISBN 978-981-02-4702-7. ^ a b Are Your Polyhedra the Same as My Polyhedra? Branko Grünbaum (2003), Fig. 3 ^ Regular polytopes, p.95 ^ Coxeter, The Densities of the Regular Polytopes II, 1932, p.53 Lee, Hwa Young; "Origami-Constructible Numbers". Coxeter, H.S.M. (1948). Regular Polytopes. Methuen and Co. Grünbaum, B.; Are your polyhedra the same as my polyhedra?, Discrete and comput. geom: the Goodman-Pollack festschrift, Ed. Aronov et al., Springer (2003), pp. 461–488. Poinsot, L.; Memoire sur les polygones et polyèdres. J. de l'École Polytechnique 9 (1810), pp. 16–48. Weisstein, Eric W. "Regular polygon". MathWorld. Regular Polygon description With interactive animation Incircle of a Regular Polygon With interactive animation Area of a Regular Polygon Three different formulae, with interactive animation Renaissance artists' constructions of regular polygons at Convergence vteFundamental convex regular and uniform polytopes in dimensions 2–10 Family An Bn I2(p) / Dn E6 / E7 / E8 / F4 / G2 Hn Regular polygon Triangle Square p-gon Hexagon Pentagon Uniform polyhedron Tetrahedron Octahedron • Cube Demicube Dodecahedron • Icosahedron Uniform polychoron Pentachoron 16-cell • Tesseract Demitesseract 24-cell 120-cell • 600-cell Uniform 5-polytope 5-simplex 5-orthoplex • 5-cube 5-demicube Uniform 6-polytope 6-simplex 6-orthoplex • 6-cube 6-demicube 122 • 221 Uniform 7-polytope 7-simplex 7-orthoplex • 7-cube 7-demicube 132 • 231 • 321 Uniform 8-polytope 8-simplex 8-orthoplex • 8-cube 8-demicube 142 • 241 • 421 Uniform 9-polytope 9-simplex 9-orthoplex • 9-cube 9-demicube Uniform 10-polytope 10-simplex 10-orthoplex • 10-cube 10-demicube Uniform n-polytope n-simplex n-orthoplex • n-cube n-demicube 1k2 • 2k1 • k21 n-pentagonal polytope Topics: Polytope families • Regular polytope • List of regular polytopes and compounds • Polytope operations Retrieved from " 2Uniform 6-polytope 122 Rectified 122 Birectified 122 [clarification needed]Trirectified 122 Truncated 122 221 Rectified 221 Orthogonal projections in E6 Coxeter plane In 6-dimensional geometry, the 122 polytope is a uniform polytope, constructed from the E6 group. It was first published in E. L. Elte's 1912 listing of semiregular polytopes, named as V72 (for its 72 vertices).[1] Its Coxeter symbol is 122, describing its bifurcating Coxeter-Dynkin diagram, with a single ring on the end of the 1-node sequence. There are two rectifications of the 122, constructed by positions points on the elements of 122. The rectified 122 is constructed by points at the mid-edges of the 122. The birectified 122 is constructed by points at the triangle face centers of the 122. These polytopes are from a family of 39 convex uniform polytopes in 6-dimensions, made of uniform polytope facets and vertex figures, defined by all permutations of rings in this Coxeter-Dynkin diagram: . 122 polytope Type Uniform 6-polytope Family 1k2 polytope Schläfli symbol {3,32,2} Coxeter symbol 122 Coxeter-Dynkin diagram or 5-faces 54:27 121 27 121 4-faces 702:270 111 432 120 Cells 2160:1080 110 1080 {3,3} Faces 2160 {3} Edges 720 Vertices 72 Vertex figure Birectified 5-simplex:022 Petrie polygon Dodecagon Coxeter group E6, [[3,32,2]], order 103680 Properties convex, isotopic The 122 polytope contains 72 vertices, and 54 5-demicubic facets. It has a birectified 5-simplex vertex figure. Its 72 vertices represent the root vectors of the simple Lie group E6. Pentacontatetrapeton (Acronym: mo) - 54-facetted polypeton (Jonathan Bowers)[2] Coxeter plane orthographic projections E6[12] D5[8] D4 / A2[6] (1,2) (1,3) (1,9,12) B6[12/2] A5[6] A4[[5]] = [10] A3 / D3[4] (1,2) (2,3,6) (1,2) (1,6,8,12) It is created by a Wythoff construction upon a set of 6 hyperplane mirrors in 6-dimensional space. The facet information can be extracted from its Coxeter-Dynkin diagram, . Removing the node on either of 2-length branches leaves the 5-demicube, 121, . The vertex figure is determined by removing the ringed node and ringing the neighboring node. This makes the birectified 5-simplex, 022, . Seen in a configuration matrix, the element counts can be derived by mirror removal and ratios of Coxeter group orders.[3] E6 k-face fk f0 f1 f2 f3 f4 f5 k-figure notes A5 ( ) f0 72 20 90 60 60 15 15 30 6 6 r{3,3,3} E6/A5 = 72*6!/6! = 72 A2A2A1 { } f1 2 720 9 9 9 3 3 9 3 3 {3}×{3} E6/A2A2A1 = 72*6!/3!/3!/2 = 720 A2A1A1 {3} f2 3 3 2160 2 2 1 1 4 2 2 s{2,4} E6/A2A1A1 = 72*6!/3!/2/2 = 2160 A3A1 {3,3} f3 4 6 4 1080 * 1 0 2 2 1 { }∨( ) E6/A3A1 = 72*6!/4!/2 = 1080 4 6 4 * 1080 0 1 2 1 2 A4A1 {3,3,3} f4 5 10 10 5 0 216 * * 2 0 { } E6/A4A1 = 72*6!/5!/2 = 216 5 10 10 0 5 * 216 * 0 2 D4 h{4,3,3} 8 24 32 8 8 * * 270 1 1 E6/D4 = 72*6!/8/4! = 270 D5 h{4,3,3,3} f5 16 80 160 80 40 16 0 10 27 * ( ) E6/D5 = 72*6!/16/5! = 27 16 80 160 40 80 0 16 10 * 27 Orthographic projection in Aut(E6) Coxeter plane with 18-gonal symmetry for complex polyhedron, 3{3}3{4}2. It has 72 vertices, 216 3-edges, and 54 3{3}3 faces. The regular complex polyhedron 3{3}3{4}2, , in C 2 {\displaystyle \mathbb {C} ^{2}} has a real representation as the 122 polytope in 4-dimensional space. It has 72 vertices, 216 3-edges, and 54 3{3}3 faces. Its complex reflection group is 3[3]3[4]2, order 1296. It has a half-symmetry quasiregular construction as , as a rectification of the Hessian polyhedron, .[4] Along with the semiregular polytope, 221, it is also one of a family of 39 convex uniform polytopes in 6-dimensions, made of uniform polytope facets and vertex figures, defined by all permutations of rings in this Coxeter-Dynkin diagram: . 1k2 figures in n dimensions Space Finite Euclidean Hyperbolic n 3 4 5 6 7 8 9 10 Coxetergroup E3=A2A1 E4=A4 E5=D5 E6 E7 E8 E9 = E ~ 8 {\displaystyle {\tilde {E}}_{8}} = E8+ E10 = T ¯ 8 {\displaystyle {\bar {T}}_{8}} = E8++ Coxeterdiagram Symmetry(order) [3−1,2,1] [30,2,1] [31,2,1] [[32,2,1]] [33,2,1] [34,2,1] [35,2,1] [36,2,1] Order 12 120 1,920 103,680 2,903,040 696,729,600 ∞ Graph - - Name 1−1,2 102 112 122 132 142 152 162 The 122 is related to the 24-cell by a geometric folding E6 → F4 of Coxeter-Dynkin diagrams, E6 corresponding to 122 in 6 dimensions, F4 to the 24-cell in 4 dimensions. This can be seen in the Coxeter plane projections. The 24 vertices of the 24-cell are projected in the same two rings as seen in the 122. E6/F4 Coxeter planes 122 24-cell D4/B4 Coxeter planes 122 24-cell This polytope is the vertex figure for a uniform tessellation of 6-dimensional space, 222, . Rectified 122 Type Uniform 6-polytope Schläfli symbol 2r{3,3,32,1}r{3,32,2} Coxeter symbol 0221 Coxeter-Dynkin diagram or 5-faces 126 4-faces 1566 Cells 6480 Faces 6480 Edges 6480 Vertices 720 Vertex figure 3-3 duoprism prism Petrie polygon Dodecagon Coxeter group E6, [[3,32,2]], order 103680 Properties convex The rectified 122 polytope (also called 0221) can tessellate 6-dimensional space as the Voronoi cell of the E6* honeycomb lattice (dual of E6 lattice).[5] Birectified 221 polytope Rectified pentacontatetrapeton (Acronym: ram) - rectified 54-facetted polypeton (Jonathan Bowers)[6] Vertices are colored by their multiplicity in this projection, in progressive order: red, orange, yellow. Coxeter plane orthographic projections E6[12] D5[8] D4 / A2[6] B6[12/2] A5[6] A4[5] A3 / D3[4] Its construction is based on the E6 group and information can be extracted from the ringed Coxeter-Dynkin diagram representing this polytope: . Removing the ring on the short branch leaves the birectified 5-simplex, . Removing the ring on either of 2-length branches leaves the birectified 5-orthoplex in its alternated form: t2(211), . The vertex figure is determined by removing the ringed node and ringing the neighboring ring. This makes 3-3 duoprism prism, {3}×{3}×{}, . Seen in a configuration matrix, the element counts can be derived by mirror removal and ratios of Coxeter group orders.[3][6] E6 k-face fk f0 f1 f2 f3 f4 f5 k-figure notes A2A2A1 ( ) f0 720 18 18 18 9 6 18 9 6 9 6 3 6 9 3 2 3 3 {3}×{3}×{ } E6/A2A2A1 = 72*6!/3!/3!/2 = 720 A1A1A1 { } f1 2 6480 2 2 1 1 4 2 1 2 2 2 1 2 4 1 1 2 2 { }∨{ }∨( ) E6/A1A1A1 = 72*6!/2/2/2 = 6480 A2A1 {3} f2 3 3 4320 * * 1 2 1 0 0 2 1 1 2 0 1 2 1 Sphenoid E6/A2A1 = 72*6!/3!/2 = 4320 3 3 * 4320 * 0 2 0 1 1 1 0 2 2 1 1 1 2 A2A1A1 3 3 * * 2160 0 0 2 0 2 0 1 0 4 1 0 2 2 { }∨{ } E6/A2A1A1 = 72*6!/3!/2/2 = 2160 A2A1 {3,3} f3 4 6 4 0 0 1080 * * * * 2 1 0 0 0 1 2 0 { }∨( ) E6/A2A1 = 72*6!/3!/2 = 1080 A3 r{3,3} 6 12 4 4 0 * 2160 * * * 1 0 1 1 0 1 1 1 {3} E6/A3 = 72*6!/4! = 2160 A3A1 6 12 4 0 4 * * 1080 * * 0 1 0 2 0 0 2 1 { }∨( ) E6/A3A1 = 72*6!/4!/2 = 1080 {3,3} 4 6 0 4 0 * * * 1080 * 0 0 2 0 1 1 0 2 r{3,3} 6 12 0 4 4 * * * * 1080 0 0 0 2 1 0 1 2 A4 r{3,3,3} f4 10 30 20 10 0 5 5 0 0 0 432 * * * * 1 1 0 { } E6/A4 = 72*6!/5! = 432 A4A1 10 30 20 10 0 5 0 5 0 0 * 216 * * * 0 2 0 E6/A4A1 = 72*6!/5!/2 = 216 A4 10 30 10 20 0 0 5 0 5 0 * * 432 * * 1 0 1 E6/A4 = 72*6!/5! = 432 D4 {3,4,3} 24 96 32 32 32 0 8 8 0 8 * * * 270 * 0 1 1 E6/D4 = 72*6!/8/4! = 270 A4A1 r{3,3,3} 10 30 0 20 10 0 0 0 5 5 * * * * 216 0 0 2 E6/A4A1 = 72*6!/5!/2 = 216 A5 2r{3,3,3,3} f5 20 90 60 60 0 15 30 0 15 0 6 0 6 0 0 72 * * ( ) E6/A5 = 72*6!/6! = 72 D5 2r{4,3,3,3} 80 480 320 160 160 80 80 80 0 40 16 16 0 10 0 * 27 * E6/D5 = 72*6!/16/5! = 27 80 480 160 320 160 0 80 40 80 80 0 0 16 10 16 * * 27 Truncated 122 Type Uniform 6-polytope Schläfli symbol t{3,32,2} Coxeter symbol t(122) Coxeter-Dynkin diagram or 5-faces 72+27+27 4-faces 32+216+432+270+216 Cells 1080+2160+1080+1080+1080 Faces 4320+4320+2160 Edges 6480+720 Vertices 1440 Vertex figure ( )v{3}x{3} Petrie polygon Dodecagon Coxeter group E6, [[3,32,2]], order 103680 Properties convex Truncated 122 polytope (Acronym: tim)[7] Its construction is based on the E6 group and information can be extracted from the ringed Coxeter-Dynkin diagram representing this polytope: . Vertices are colored by their multiplicity in this projection, in progressive order: red, orange, yellow. Coxeter plane orthographic projections E6[12] D5[8] D4 / A2[6] B6[12/2] A5[6] A4[5] A3 / D3[4] Birectified 122 polytope Type Uniform 6-polytope Schläfli symbol 2r{3,32,2} Coxeter symbol 2r(122) Coxeter-Dynkin diagram or 5-faces 126 4-faces 2286 Cells 10800 Faces 19440 Edges 12960 Vertices 2160 Vertex figure Coxeter group E6, [[3,32,2]], order 103680 Properties convex Bicantellated 221 Birectified pentacontatetrapeton (barm) (Jonathan Bowers)[8] Vertices are colored by their multiplicity in this projection, in progressive order: red, orange, yellow. Coxeter plane orthographic projections E6[12] D5[8] D4 / A2[6] B6[12/2] A5[6] A4[5] A3 / D3[4] Trirectified 122 polytope Type Uniform 6-polytope Schläfli symbol 3r{3,32,2} Coxeter symbol 3r(122) Coxeter-Dynkin diagram or 5-faces 558 4-faces 4608 Cells 8640 Faces 6480 Edges 2160 Vertices 270 Vertex figure Coxeter group E6, [[3,32,2]], order 103680 Properties convex Tricantellated 221 Trirectified pentacontatetrapeton (Acronym: trim, old: cacam, tram, mak) (Jonathan Bowers)[9] List of E6 polytopes ^ Elte, 1912 ^ Klitzing, (o3o3o3o3o *c3x - mo) ^ a b Coxeter, Regular Polytopes, 11.8 Gosset figures in six, seven, and eight dimensions, pp. 202–203 ^ Coxeter, H. S. M., Regular Complex Polytopes, second edition, Cambridge University Press, (1991). p.30 and p.47 ^ The Voronoi Cells of the E6* and E7* Lattices Archived 2016-01-30 at the Wayback Machine, Edward Pervin ^ a b Klitzing, (o3o3x3o3o *c3o - ram) ^ Klitzing, (o3o3o3x3o *c3x - tim) ^ Klitzing, (o3x3o3x3o *c3o - barm) ^ Klitzing, (x3o3o3o3x *c3o - trim) Elte, E. L. (1912), The Semiregular Polytopes of the Hyperspaces, Groningen: University of Groningen H. S. M. Coxeter, Regular Polytopes, 3rd Edition, Dover New York, 1973 Kaleidoscopes: Selected Writings of H.S.M. Coxeter, edited by F. Arthur Sherk, Peter McMullen, Anthony C. Thompson, Asia Ivic Weiss, Wiley-Interscience Publication, 1995, wiley.com, ISBN 978-0-471-01003-6 (Paper 24) H.S.M. Coxeter, Regular and Semi-Regular Polytopes III, [Math. Zeit. 200 (1988) 3-45] See p334 (figure 3.6a) by Peter McMullen: (12-gonal node-edge graph of 122) Klitzing, Richard. "6D uniform polytopes (polypeta) with acronyms". o3o3o3o3o *c3x - mo, o3o3x3o3o *c3o - ram, o3x3o3o3o *c3x - tim, o3x3o3x3o *c3o - barm, x3o3o3o3x *c3o - trim vteFundamental convex regular and uniform polytopes in dimensions 2–10 Family An Bn I2(p) / Dn E6 / E7 / E8 / F4 / G2 Hn Regular polygon Triangle Square p-gon Hexagon Pentagon Uniform polyhedron Tetrahedron Octahedron • Cube Demicube Dodecahedron • Icosahedron Uniform polychoron Pentachoron 16-cell • Tesseract Demitesseract 24-cell 120-cell • 600-cell Uniform 5-polytope 5-simplex 5-orthoplex • 5-cube 5-demicube Uniform 6-polytope 6-simplex 6-orthoplex • 6-cube 6-demicube 122 • 221 Uniform 7-polytope 7-simplex 7-orthoplex • 7-cube 7-demicube 132 • 231 • 321 Uniform 8-polytope 8-simplex 8-orthoplex • 8-cube 8-demicube 142 • 241 • 421 Uniform 9-polytope 9-simplex 9-orthoplex • 9-cube 9-demicube Uniform 10-polytope 10-simplex 10-orthoplex • 10-cube 10-demicube Uniform n-polytope n-simplex n-orthoplex • n-cube n-demicube 1k2 • 2k1 • k21 n-pentagonal polytope Topics: Polytope families • Regular polytope • List of regular polytopes and compounds • Polytope operations Retrieved from " 3 The following pages link to 1 22 polytope External tools (link count transclusion count sorted list) · See help page for transcluding these entries Showing 50 items. View (previous 50 | next 50) (20 | 50 | 100 | 250 | 500)Dodecahedron (links | edit) Regular icosahedron (links | edit) Regular octahedron (links | edit) Polytope (links | edit) Polygon (links | edit) 4-polytope (links | edit) Tetrahedron (links | edit) Tesseract (links | edit) Simplex (links | edit) Hypercube (links | edit) Hexagon (links | edit) Equilateral triangle (links | edit) List of mathematical shapes (links | edit) Root system (links | edit) 24-cell (links | edit) E6 (mathematics) (links | edit) Coxeter group (links | edit) Regular polygon (links | edit) 72 (number) (links | edit) 600-cell (links | edit) List of regular polytopes (links | edit) Dodecagon (links | edit) 5-cell (links | edit) List of polygons, polyhedra and polytopes (links | edit) Regular polytope (links | edit) Square (links | edit) Cross-polytope (links | edit) 16-cell (links | edit) 120-cell (links | edit) Point group (links | edit) Uniform 4-polytope (links | edit) Duoprism (links | edit) Coxeter element (links | edit) Runcinated 5-cell (links | edit) Runcinated tesseracts (links | edit) Uniform polyhedron (links | edit) Rectified 600-cell (links | edit) Rectified 5-cell (links | edit) Snub 24-cell (links | edit) Cantellated tesseract (links | edit) Truncated tesseract (links | edit) Truncated 5-cell (links | edit) Grand antiprism (links | edit) Rectified 120-cell (links | edit) Rectified 24-cell (links | edit) Cantellated 5-cell (links | edit) Cantellated 24-cells (links | edit) Runcinated 24-cells (links | edit) Truncated 120-cells (links | edit) View (previous 50 | next 50) (20 | 50 | 100 | 250 | 500) Retrieved from " WhatLinksHere/1_22_polytope" Polygon Properties (Math | Geometry | Polygon Properties) What is a Polygon? A closed plane figure made up of several line segments that are joined together. The sides do not cross each other. Exactly two sides meet at every vertex. Types | Formulas | Parts | Special Polygons | Names Types of Polygons Regular - all angles are equal and all sides are the same length. Regular polygons are both equiangular and equilateral. Equiangular - all angles are equal. Equilateral - all sides are the same length. Convex - a straight line drawn through a convex polygon crosses at most two sides. Every interior angle is less than 180°. Concave - you can draw at least one straight line through a concave polygon that crosses more than two sides. At least one interior angle is more than 180°. Polygon Formulas (N = # of sides and S = length from center to a corner) Area of a regular polygon = (1/2) N sin(360°/N) S2 Sum of the interior angles of a polygon = (N - 2) x 180° The number of diagonals in a polygon = 1/2 N(N-3) The number of triangles (when you draw all the diagonals from one vertex) in a polygon = (N - 2) Polygon Parts Side - one of the line segments that make up the polygon. Vertex - point where two sides meet. Two or more of these points are called vertices. Diagonal - a line connecting two vertices that isn't a side. Interior Angle - Angle formed by two adjacent sides inside the polygon. Exterior Angle - Angle formed by two adjacent sides outside the polygon. Special Polygons Special Quadrilaterals - square, rhombus, parallelogram, rectangle, and the trapezoid. Special Triangles - right, equilateral, isosceles, scalene, acute, obtuse. Polygon Names Generally accepted names Sides Name n N-gon 3 Triangle 4 Quadrilateral 5 Pentagon 6 Hexagon 7 Heptagon 8 Octagon 10 Decagon 12 Dodecagon Names for other polygons have been proposed. Sides Name 9 Nonagon, Enneagon 11 Undecagon, Hendecagon 13 Tridecagon, Triskaidecagon 14 Tetradecagon, Tetrakaidecagon 15 Pentadecagon, Pentakaidecagon 16 Hexadecagon, Hexakaidecagon 17 Heptadecagon, Heptakaidecagon 18 Octadecagon, Octakaidecagon 19 Enneadecagon, Enneakaidecagon 20 Icosagon 30 Triacontagon 40 Tetracontagon 50 Pentacontagon 60 Hexacontagon 70 Heptacontagon 80 Octacontagon 90 Enneacontagon 100 Hectogon, Hecatontagon 1,000 Chiliagon 10,000 Myriagon To construct a name, combine the prefix+suffix Sides Prefix 20 Icosikai... 30 Triacontakai... 40 Tetracontakai... 50 Pentacontakai... 60 Hexacontakai... 70 Heptacontakai... 80 Octacontakai... 90 Enneacontakai... + Sides Suffix +1 ...henagon +2 ...digon +3 ...trigon +4 ...tetragon +5 ...pentagon +6 ...hexagon +7 ...heptagon +8 ...octagon +9 ...enneagon Examples: 46 sided polygon - Tetracontakaihexagon 28 sided polygon - Icosikaioctagon However, many people use the form n-gon, as in 46-gon, or 28-gon instead of these names. These are the properties of a regular polygon: The sides and interior angles of a regular polygon are all equal. The bisectors of the interior angles of a regular polygon meet at its center. The perpendiculars drawn from the center of a regular polygon to its sides are all equal. The lines joining the center of a regular polygon to its vertices are all equal. The center of a regular polygon is the center of both the inscribed and circumscribed circles. Straight lines drawn from the center to the vertices of a regular polygon divide it into as many equal isosceles triangles as there are sides in it. The angle of a regular polygon of $$n$$ sides $$ = \left({\frac{{2n – 4}}{n}} \right) \times {90^ \circ }$$. Detail of the Sides of a Polygon There is no theoretical limit to the number of sides of a polygon, but only those with twelve or less are commonly seen. The names of polygons which are used most often are as follows: Number of sides Polygon name $$5$$ Pentagon $$6$$ Hexagon $$7$$ Heptagon $$8$$ Octagon $$9$$ Nonagon $$10$$ Decagon $$ \cdots $$ $$ \cdots $$ $$ \cdots $$ $$ \cdots $$ $$n$$ $$n – $$gon Example: The perimeter and area of a regular polygon are respectively equal to those of a square of sides $$a$$. Find the length of the perpendicular from the center of a regular polygon to any of its sides. Solution: The perimeter of a regular polygon = perimeter of square $$ = 4a$$ A regular polygon can be divided into congruent triangles having a common vertex at the center of the polygon. The number of these congruent triangles is the same as that of its sides. $$\therefore $$ the area of one such triangle $$ = \frac{1}{2} \times $$ sides of polygon $$ \times $$ length of perpendicular from the center to any side of polygon. $$\therefore $$ the area of one such triangle$$ = \frac{1}{2}ah$$, $$h$$ being the length of the perpendicular $$\therefore $$ Area of polygon = Sum of areas of all such triangles $$\therefore $$ Area of polygon $$ = \frac{1}{2} \times $$ Perimeter of polygon $$ \times {\text{ }}h$$ $$\therefore $$ Area of polygon $$ = \frac{1}{2} \times 4a \times h = 2ah$$ — (1) Now, the area of the polygon = Area of square$$ = {a^2}$$ (given) — (2) $$\therefore $$ from (1) and (2), we have $$\therefore $$ $$2ah = {a^2} \Rightarrow h = \frac{a}{2}$$ Hence, the length of the perpendicular is $$\frac{a}{2}$$. Written by January 16, 2023Fact-checked by Paul MazzolaPolygonRegular polygonPropertiesNamesPartsAnglesFormulasPolygons are straight-sided, flat shapes that close in a space. This means every polygon:Is two-dimensionalUses only line segments to create its sidesHas an interior and exteriorThe word "polygon" comes from the Greek and means "many angles," because all simple polygons have as many sides as they have angles. The simplest polygon, the triangle, has three interior angles (and three exterior angles!) so it also has three sides.You can have polygons with so many sides, they appear to be circles even when they are not. Once you get above 10 sides, most mathematics students just say "n-gon," where n is the number of sides or interior angles, as in a "18-gon," or a "25-gon," rather than trying to remember the Greek-origin name.We have spoken before about polygons, but we have not tackled regular polygons by themselves. They are the crème de la crème of polygons, a class unto themselves!Get free estimates from geometry tutors near you.What is a regular polygon?To be a regular polygon, the flat, closed, straight-sided shape must also have another property. Every interior and exterior angle in the regular polygon is equal to every other interior and exterior angle, and every side is equal in length to every other side.Regular Polygon DefinitionA regular polygon has:Two dimensionsStraight sidesCongruent (equal-length) sidesAn interior and exteriorEqual interior anglesEqual exterior anglesRegular polygon examplesA few examples of regular polygons are a triangle, quadrilateral, pentagon, hexagon, heptagon, and a decagon.Regular polygon shapesIf even one of those features is not present, you do not have a regular polygon. You may have three of the features (two dimensions; straight sides; an interior and exterior) but still not have a regular polygon. You would have an irregular polygon.Irregular polygon shapesA rectangle that is longer than it is tall is an example of an irregular polygon. A scalene triangle, home plate on a baseball or softball field, and a kite are all also examples of irregular polygons.Properties of regular polygonsOnly when all six conditions, outlined above, are present will you have a regular polygon. Let's examine a suspicious shape and see:Here we have △CAT, drawn with black lines. So our black CAT is suspicious, indeed.Properties of Regular PolygonsGo through the six properties of regular polygons:Is it two-dimensional?Is it made with straight sides?Are its sides congruent?Does it close in a space, creating an interior and exterior?Does it have equal interior angles?Does it have equal exterior angles?Results of our scrutiny using the six identifying properties of regular polygons: △CAT is a regular polygon, the simplest kind. It is an equilateral triangle, with interior angles of 60° and exterior angles of 120°.Names of regular polygonsImagine you are unpacking a shipment of polygons. You have to put all the irregular polygons on the discount shelf, and all the regular polygons on the full-price shelf.Using your knowledge of the identifying properties of regular polygons, you can easily see that only a few typical polygons are regular: Regular Polygon Number of Sides Interior Angles Exterior Angles Equilateral triangle 3 sides 3 interior angles of 60° 3 exterior angles of 120° Square 4 sides 4 interior angles of 90° 4 exterior angles of 90° Regular pentagon 5 sides 5 interior angles of 108° 5 exterior angles of 72° Regular hexagon 6 sides 6 interior angles of 120° 6 exterior angles of 60° Regular heptagon 7 sides 7 interior angles ≅ 128.57° 7 exterior angles ≅ 51.43° Regular octagon 8 sides 8 interior angles of 135° 8 exterior angles of 45° Names Of Regular PolygonYou can create regular polygons of any number of sides and interior angles, but usually you will not name them with anything other than the number of their sides or interior angles and then -gon:Number of sides for various polygons Regular Polygon Number of Sides Interior Angles Exterior Angles Regular enneagon or 9-gon 9 sides 9 interior angles of 140° 9 exterior angles of 40° Regular decagon or 10-gon 10 sides 10 interior angles of 144° 10 exterior angles of 36° Regular hectogon or 100-gon 100 sides 100 interior angles of 176.4° 100 exterior angles of 3.6° Get free estimates from geometry tutors near you.Any polygon that does not have equal-length sides along with equal interior and exterior angles is an irregular polygon; it goes on the discount shelf!Parts of regular polygonsAs simple as they are, regular polygons still have six parts:SidesInterior and exteriorInterior anglesExterior anglesVerticesDiagonalsWe will illustrate with a basic regular polygon, the hexagon. This is a familiar shape (think of the cells of a honeybee hive):Parts of a Regular PolygonOur hexagon has six congruent sides. It has six equal interior angles, each 120°120°. It also has six equal exterior angles. An exterior angle is created by extending one side and measuring the angle between that extension and an adjacent side.Regular polygon anglesExterior angles of a regular polygonExterior angles of every simple polygon add up to 360°, because a trip around the polygon completes a rotation, or return to your starting place. Where sides meet, they form vertices, so our hexagon also has six vertices.Interior angles of a regular polygonInside the hexagon's sides, where the interior angles are, is the hexagon's interior. Outside its sides is the hexagon's exterior. This becomes important when you consider complex polygons, like a star-shape (a pentagram, for example).Get free estimates from geometry tutors near you.Diagonal of a polygonConnect any non-adjacent vertices in the interior of the hexagon to make diagonals. All simple polygons can be divided into triangles using diagonals. The minimum number of triangles created in our hexagon, by drawing three diagonals, is four; each triangle has interior angles adding to 180°, so the sum of the interior angles of a regular hexagon is 4 × 180°, or 720°.Regular polygon formulaThe formula for calculating the sum of interior angles of any regular polygon with n sides is:To find any single interior angle, divide your answer by n.Lesson summaryNow you have all the tools you need to identify polygons, tell regular ones from irregular ones, name regular polygons and recognize them by their properties, list the parts of regular polygons (sides, interior, interior angles, vertices, diagonals, exterior angles), and calculate the sums of the interior angles of polygons.After viewing the video and studying the drawings, please read the entire lesson. Then you will learn to:Identify polygonsIdentify regular and irregular polygonsState the identifying properties of regular polygonsName examples of regular polygonsList six parts of regular polygonsCalculate the sum of interior angles of regular polygons A two-dimensional enclosed figure made by joining three or more straight lines is known as a polygon. They are also known as "flat figures". Example: A square is a polygon with made by joining 4 straight lines of equal length. A polygon has three parts: Sides: A line segment that joins two vertices is known as a side. Vertices: The point at which two sides meet is known as a vertex. Angles: interior and exterior. An interior angle is the angle formed within the enclosed surface of the polygon by joining the sides. If all the polygon sides and interior angles are equal, then they are known as regular polygons. The examples of regular polygons are square, equilateral triangle, etc. In regular polygons, not only are the sides congruent but so are the angles. That means they are equiangular. The properties of regular polygons are listed below: All its sides are equal. All its interior angles are equal. The sum of its exterior angles is 360°. A regular polygon has all the sides equal. And the perimeter of a polygon is the sum of all the sides. So, a regular polygon with n sides has the perimeter = n times of a side measure. For example, if the side of a regular polygon is 6 cm and the number of sides are 5, perimeter = 5 × 6 = 30 cm Let there be a n sided regular polygon. Since, the sides of a regular polygon are equal, the sum of interior angles of a regular polygon = (n − 2) × 180° For example, the sides of a regular polygon are 6. So, the sum of interior angles of a 6 sided polygon = (n − 2) × 180° = (6 − 2) × 180° = 720° Since a regular polygon is equiangular, the angles of n sided polygon will be of equal measure. A n sided polygon has each interior angle = $\frac{Sum of interior angles}{n}$$=$$\frac{(n-2)\times180^\circ}{n}$ For example, let's take a regular polygon that has 8 sides. So, each interior angle = $\frac{(8-2)\times180^\circ}{8} = 135^\circ$ There are n equal angles in a regular polygon and the sum of an exterior angles of a polygon is 360°. So, the measure of each exterior angle of a regular polygon = $\frac{360^\circ}{n}$. The number of diagonals in a polygon with n sides = $\frac{n(n-3)}{2}$ as each vertex connects to (n – 3) vertices. And in order to avoid double counting, we divide it by two. For example, if the number of sides of a regular regular are 4, then the number of diagonals = $\frac{4\times1}{2}=2$. If the sides of a regular polygon are n, then the number of triangles formed by joining the diagonals from one corner of a polygon = n – 2 For example, if the number of sides are 4, then the number of triangles formed will be (4 – 2) = 2. The line of symmetry can be defined as the axis or imaginary line that passes through the center of the shape or object and divides it into identical halves. Since all the sides of a regular polygon are equal, the number of lines of symmetry = number of sides = n For example, a square has 4 sides. So, the number of lines of symmetry = 4 A shape has rotational symmetry when it can be rotated and still it looks the same. The order of a rotational symmetry of a regular polygon = number of sides = n . Also, the angle of rotational symmetry of a regular polygon = $\frac{360^\circ}{n}$. For example, a square has 4 sides. So, the order of rotational symmetry = 4. Angle of rotation =$\frac{360}{4}=90^\circ$. This means when we rotate the square 4 times at an angle of 90°, we will get the same image each time. The following is a list of regular polygons: A circle is a regular 2D shape, but it is not a polygon because it does not have any straight sides. Example 1: Find the number of diagonals of a regular polygon of 12 sides. Solution: The number of diagonals of a n sided polygon = $n\frac{(n-3)}{2}$$=$$12\frac{(12-3)}{2}=54$ Example 2: If each interior angle of a regular polygon is 120°, what will be the number of sides? Solution: We know that each interior angle = $\frac{(n-2)\times180^\circ}{n}$, where n is the number of sides. So, $120^\circ$$=$$\frac{(n-2)\times180^\circ}{n}$ $120n=(n-2)$$\times$$180$ $120n=180n-360$ $n=6$ The number of sides = 6 Example 3: Can a regular polygon have an internal angle of 100° each? Solution: Each exterior angle = $180^\circ – 100^\circ = 80^\circ$ Each exterior angles = $\frac{360^\circ}{n}$, where n is the number of sides. 80° = $\frac{360^\circ}{n}$$\Rightarrow$ n = 4.5, which is not possible as the number of sides can not be in decimal. Attend this quiz & Test your knowledge.It has (n - 2) triangles.It has (n - 3) lines of symmetry.Its interior angle is $\frac{(n-2)180^\circ}{n}$Correct answer is: It has (n - 3) lines of symmetry.A regular polygon has n lines of symmetry.Correct answer is: 1260°$n=9$ in a regular nonagon. So, sum of interior angles = $(n-2)\times180° =7\times180°=1260°$Correct answer is: 18Each exterior angle = 20° . Number of sides = $\frac{360°}{Each exterior angle}=\frac{360°}{20}=18$. What is the difference between a regular and an irregular polygon? A regular polygon is a polygon that is equilateral and equiangular, such as square, equilateral triangle, etc. On the other hand, an irregular polygon is a polygon that does not have all sides equal or angles equal, such as a kite, scalene triangle, etc. Why is a rhombus not a regular polygon? A rhombus is not a regular polygon because the opposite angles of a rhombus are equal and a regular polygon has all angles equal. How to find the sides of a regular polygon if each exterior angle is given? The side of regular polygon = $\frac{360^\circ}{Each exterior angle}$ A polygon is a plane shape (two-dimensional) with straight sides. Polygons are all around us, from doors and windows to stop signs. Let's explore what makes some of them 'regular' and why that matters! Regular A regular polygon has: all sides equal and all angles equal. Otherwise it is irregular. Regular Pentagon Irregular Pentagon Regular polygons have some cool properties, let's explore them. Properties of Regular Polygons So what can we know about regular polygons? First of all, we can work out angles. The Exterior Angle is the angle between any side of a shape, and a line extended from the next side. images/exterior-angles.js All the exterior angles of a polygon add up to 360°, so: Each exterior angle must be 360°/n (where n is the number of sides) Press play button to see. Imagine walking around the edge of a building, turning at each corner. By the time you return to where you started, you've turned a full 360° Exterior Angle (of a regular octagon) An octagon has 8 sides, so: Exterior angle =360° / n =360° / 8 =45° Interior Angles The Interior Angle and Exterior Angle are measured from the same line, so they add up to 180°. Interior Angle = 180° − Exterior Angle Since we already know the Exterior Angle is 360°/n: Interior Angle = 180° − 360°/n Which can be rearranged like this: Interior Angle= 180° − 360°/n = (n × 180° / n) − (2 × 180° / n) = (n−2) × 180°/n So we also have this: Interior Angle = (n−2) × 180° / n A regular octagon has 8 sides, so: Exterior Angle = 360° / 8 = 45° Interior Angle = 180° − 45° = 135° Interior Angle (of a regular octagon) Or we could use: Interior Angle= (n−2) × 180° / n = (8−2) × 180° / 8 = 6 × 180° / 8 = 135° Same answer. A regular hexagon has 6 sides, so: Exterior Angle = 360° / 6 = 60° Interior Angle = 180° − 60° = 120° Some polygons have special circles hidden inside and around them! Let's explore them and their names: "Circumcircle, Incircle, Radius and Apothem ..." Sounds quite musical if you repeat it a few times, but they are just the names of the "outer" and "inner" circles (and each radius) that can be drawn on a polygon like this: The "outside" circle is called a circumcircle, and it connects all vertices (corner points) of the polygon. The radius of the circumcircle is also the radius of the polygon. The "inside" circle is called an incircle and it just touches each side of the polygon at its midpoint. The radius of the incircle is the apothem of the polygon. (Not all polygons have those properties, but all triangles and regular polygons do). Breaking into Triangles We can learn a lot about regular polygons by breaking them into triangles like this: Notice that: the "base" of the triangle is one side of the polygon. the "height" of the triangle is the "Apothem" of the polygon Now, the area of a triangle is half of the base times height, so: Area of one triangle = base × height / 2 = side × apothem / 2 To get the area of the whole polygon, just add up the areas of all the little triangles ("n" of them): Area of Polygon = n × side × apothem / 2 And since the perimeter is all the sides = n × side, we get: Area of Polygon = perimeter × apothem / 2 A Smaller Triangle By cutting the triangle in half we get this: (Note: The angles are in radians, not degrees) The small triangle is right-angled and so we can use sine, cosine and tangent to find how the side, radius, apothem and n (number of sides) are related: sin(π/n) = (Side/2) / Radius Side = 2 × Radius × sin(π/n) cos(π/n) = Apothem / Radius Apothem = Radius × cos(π/n) tan(π/n) = (Side/2) / Apothem Side = 2 × Apothem × tan(π/n) There are a lot more relationships like those (most of them just "re-arrangements"), but these are the most useful ones. 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