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Differential equation with coefficient linear in x and y. The differential equation (x+2 y 3)dy dx=y is linear in unknown−−−.

Classification of differential equations We remind you that a differential equation is an equation (it has an equal sign) that involves derivatives. As biologists have a classification system for life, mathematicians have a classification system for differential equations. We can insert all differential equations in two types: ordinary differential equation and partial differential equations. A partial differential equation is a differential equation that involves partial derivatives. An ordinary differential equation is a differential equation that does not involve partial derivatives. Examples D2Y DY + = 3xsin y DX2 DX is an ordinary differential equation as it does not contain partial derivatives. It is a partial differential equation, since Y is a function of the two X and T variables and partial derivatives are present. In this course we will concentrate only on ordinary differential equations. Another way to classify differential equations is by order. Any ordinary differential equation can be written in form f (x, y, y ', y', ..., y (n) = 0 setting all equal to zero. The order of a differential equation is the most derivative High that appears in the equation above. Examples D2Y DY + = 3xsin y DX2 DX is a second order differential equation, since a second derivative appears in the equation. 3Y4Y '' '- X3Y' + Exyy = 0 Â ¨ Third order differential equation. Once we have written a differential equation in form f (x, y, y ', y', y (n) = 0 we can talk if a differential equation is linear or not . Let's say that the differential equation above is a linear differential equation if for all IIE j. Any linear regular differential equation of grade N can be written as A0 (x) y (n) + a1 (x) y (n- 1) + ... + an-1 (x) y '+ an (x) y = g (x) y = g (x) examples 3x2y' + 2ln (x) y '+ ex y = 3xcos x is A second-order linear differential equation. 4YY '' '- X3Y' + COS Y = E2x is not u The linear differential equation due to the 4YYY '' 'and terms COS y. Non-linear differential equations are often very difficult or impossible to solve. An approach around this difficulty is to linearize the differential equation. Example Y '' + 2Y '+ EY = X is not linear due to the term EY. However EY = 1 + Y + Y2 / 2 + Y3 / 6 + ... we can approximate this of 1 + Y instead solve the linear differential equation much easier Y '' '+ 2y' + 1 + y = x we say that f (x) is a solution to a differential equation if it connects f (x) in the equation makes the equation equal. Example Show Thatâ Â Â Â Â Â Â f (x) = x + e2xÂ is a TOA solution Â - 2y'= -2 Solution Â Is the solution unique? In the example above, the answer to the first question is yes since we have verified that, Â Â f (x) = x + E2x is a solution. However, the answer to the second question is no. It may be verified that Â Â Â Â Â s (x) = Â 4 + x is also a solution. Back to the first order Differential equations Home Page Back to the differential equations Home Page Back to the mathematical department E-mail Home Page Questions and Tips The learning objectives write a first-order linear differential equation in the standard form. Find a integration factor and use it to solve a linear differential equation of first order. Solve the problems applied involving linear differential equations of the first order. Previously, we studied an application of a differential equation of the first order that involved the solution for the speed of an object. In particular, if a ball is thrown upward with an initial speed of \ (v_0 \) ft / s, then an initial value problem that describes the ball velocity after \ (t \) seconds is given by \ [\ dfrac {DV} {DT} = Â'32 \] with \ (v (0) = v_0. \) This model presupposes that the only force acting on the ball is gravity. Now we add to the problem allowing the possibility of acting air on the ball. Air resistance always acts in the opposite direction of movement. Therefore, if an object is increasing, acts of air resistance in a downward direction. If the object is falling, air resistance acts act upwards (figure \ (\ PageDex {1} \)). There is no exact relationship between the speed of an object and the air resistance acting on it. For very small objects, air resistance is proportional to speed; That is, the force due to air resistance is numerically equal to some constant times \ (k \) \ (v \). For larger objects (e.g., baseball size), depending on the shape, air resistance can be roughly proportional to the square of speed. In fact, air resistance may be proportional to \ (v ^ {1.5} (v) \) or some other \ power (v \). Figure \ (\ PageNex {1} \): forces acting on a baseball ingravity acts in a downward direction and in an air resistance acts in a direction opposite the direction of movement. We will work with linear approximation for air resistance. if we assume \ (k> 0 \), then the expression for force \ (f_a \) due to air resistance is given by \ (fa_ = â'kv \). therefore the sum of the forces acting on the object is equal toSum of gravitational force and force due to air resistance. This, in turn, is the same as the mass of the object multiplied by its acceleration in time (T) (second law of Newton). This gives us the differential equation [M DFRAC {DV} {dt} = Ã ¢ 'kvÃ ¢' mg. Finally, we impose an initial condition (V (0) = V_0,) where (V_0) is the initial speed measured in meters per second. This makes it (g = 9.8 m / s ^ 2. v (0) = v_0.) The differential equation in this problem of the initial value is an example of a first order linear differential equation. (Remember that a differential equation is the first order if the highest derivative that appears in the equation is (1).) In this section, we study linear equations of the first order and examine a method to find one General solution to these types of equations, as well as solving the problems of initial value that involve them. Definition: Differential equation of the first linear order A differential equation of the first order is linear if it can be written in the form [a (x) y Ã, â² + b (x) y = c (x), where (A (X), B (X),) and (C (X) are arbitrary functions of (x). Remember that the unknown function (y) depends on the variable (x); ie, (x) is the independent variable and (y) is the dependent variable. Some examples of first order linear differential equations are [3x ^ 2ã, '4) Y' + (xÃ ¢ '3) y = sin x] [(sin x) y'Ã ¢' (COS X) Y = Cut X] [4xy '+ (3 LN X) y = x ^ 3Ã,X. Examples of non-linear differential equations of the first order include [(Y ') ^ 4' (Y ') ^ 3 = (3xÃ ¢' 2) (Y + 4) [4Y '+ 3Y ^ 3 = 4xÃ ¢ '5] ^ [(y') ^ 2 = sin y + so x. These equations are non-linear due to terms such as ((y ã, Â²) ^ 4, y ^ 3,) etc. Because of these terms, it is impossible to put these equations in the same form as the equation. Consider the differential equation [3x ^ 2Ã ¢ '4) y - y Ã, â² + (xÃ ¢' 3) y = sin x. of solution for equations of this module. It is useful to have the coefficient of (Y Ã, Ã,) to be equal to (1). To do what happens, we divide both sides of (3x ^ 2Ã ¢ '4.) [yÃ ¢ â,¬ â² + left (dPrac {xÃ ¢' 3} {3x ^ 2Ã ¢ '4} Right) y = dfrac {sin x} {3x ^ 2Ã ¢ '4} This is called the standard form of differential equation. We will use it later when you find the solution to a first order linear differential equation. Returning to the equation, we can divide both sides of the equation from (A (X)). This leads to the equation [Y â² + DFRAC {B (X)} {A (X)} y = dfrac {c (x)} {a (x)}. Label {EQ5}} Now Define [P (x) = dPrac {B (x)} {a (x)}} and [q (x) = dfrac {c (x)} {a (x)} Then equation ref {eq5} becomes [y ... Â² + (x) y = q (x). \] We can write any first-order linear differential equation in this module, and this is indicated as the standard module for a first-order linear differential equation. Example \ (\ PageDex {1} \): Write first-order linear equations in the standard module Put each of the following linear differential equations in the standard module. Identifying \ (P (x) \) and \ (q (x) \) for each each(Y '= 3XÂ'4Y) (DFRAC {3xy} {4YÂ'3} = 2) (here (x> 0) (y = 3y'â'4x ^ 2 + 5) Solution a. Add (4y) to both sides: (Y '+ 4Y = 3x) in this equation, (x) = 4 and q (x) = 3x. B. Multiply Both sides of (4yâ3,) then subtract (8y) from each side: (DFRAC {3xy '} {4YÂ'3} = 2) (3xy' = 2 (4yâ '3) (3xy' = 6) (3xy'â'88y = 6) Finally, divide both sides for (3x) to make the coefficient of (1): {8} {3x} y = Â 'dPrac {2} {3x. (x> 0.) (if (x = 0) then the original equation becomes (0 = 2,) which is clearly a false declaration.) In this equation, (p (x) = Â 'dPrac {8} {3x and (q (x) = Â' dPrac {2} {3x. c. subtrat (y) on each side and add (4x ^ 2â ' 5): (3Y'â'Y = 4x ^ 2Â'5.) Divide the two sides of (3) DFRAC {4} {3} x ^ 2â 'dPrac {5} {3.) In this equation, (p (x) = Â' dPrac {1} {3 and (q (x) = DFRAC {4} {3} x ^ 2Â ' DFRAC {5} {3. Exercise (pageIndex {1}) Put the equation (DFRAC {(x + 3) y} {2xÂ'3yÂ'4} = 5 in standard form and identify (x) and (q (x Standard shape of a first order linear differential equation: [Y '+ P (x) y = q (x. label {deq1}] The first deadline on the left side of the equation is the derivative of the unknown function, e The second term is the product of a function known with the unknown function. This somehow remembers the power rule. If we multiply the equation ref {DEQ1} from a function still to be determined (Î¼ (X,) Then the equation becomes [Î¼ (X) Yâ € ² + Î¼ (X) P (X) Y = Î¼ (X) Q (X) q (x). The left side equation ref {deq2} can be Combined perfectly to the product rule: [DFRAC {D} {DX} [F (X) G (x)] = f (x) g (x) g (x) + f (x) gâ € ² (x). The correspondence term by term dÃ (y = f (x), g (x) = Î¼ (X) P (X) = Î¼ (X) P (X). Taking the derivative of (g (x) = Î¼ (x and setting it equal to the right side of (x) = Î¼ (X) P (X) P (X port A [Î¼â € ² (x) = Î¼ (x) p (x). This is a separable differential equation and separable for Î¼ (x.) We know (x) because it appears in the differential equation that we are solving. Change the variables e Integrate yields [[4]} Â «DFRAC {Î¼â € ² (X)} {Î¼ (X = P (X) [4PT] Â« Â «DFRAC {Î¼â € ² (X) } {Î¼x = Â «here (C_2) can be an arbitrary constant (positive or negative). This leads to a general method to resolve a first order linear differential equation. First we multiply both sides of the Equation from the integration factor (Î¼ (x) this dÃ l the left side of the equation \ref{Deq5} can be rewritten as \(\dfrac{d}{dx}(μ(x)y)\). \[\dfrac{d}{dx}(μ(x)y)=μ(x)q(x).) \[y=\dfrac{1}{μ(x)}\left[∫μ(x)q(x)dx+C\right]. onumber \] since \(μ(x)\)\) has been calculated previously, we are now finished. an important note on the constant integration (c:) may seem that we are inconsistent in the or of constant integration. However, the integral involving \(p(x)\)\) is necessary to find an integration factor for the equation. only one factor of integration is necessary to solve the equation; therefore, it is sure to assign a value for (c) for this integral. we have chosen \(C=0\). when calculating the integral within the brackets in the equation, it is necessary to keep our options open for the value of the integration constant, because our goal is to find a general family of solutions to the equation. this integration factor only guarantees this. problem-solving strategy: solve a linear differential equation of first order put equation in standard form and identify \(p(x)\)\)\) and \(q(x)\). calculates the integration factor \[μ(x)=e^{∫p(x)dx}.\] multiply both sides of the differential equation of \(μ(x)\). integrate both sides of the equation obtained in the \(3,)\ passage and divide both sides from \(μ(x)\). if there is an initial condition, determine the value of (c.) example \(\PageIndex{2}\): solve a linear equation of first order find a general solution for the differential equation \(xy'+3y=4x^2−3x.\) take \(x>0.\) solution 1. to put this differential equation in standard form, divide both sides from \(x:)\ \[y'+\dfrac{3}{x}y=4x−3. suumber]\ therefore, \(p(x)=\dfrac{3}{x}\) and \(q(x)=4x−3.\) 2. the integration factor is \(μ(x)=e^{∫(3/x)}dx=e^{3 \ln x}=x^3\). 3. multiplying on both sides of the differential equation of \(μ(x)\) gives us \[\begin{align*} x^3y\x^3(\dfrac{3}{x}) =x^3(4x−3) [\4p] x3y=3x^2y =4x^4=3 4. integrate both sides of the equation. > there is no initial value, so the problem is complete. analysis you may have noticed the condition that it isset on differential equation; i.e., \(x>0\). for any non-zero value of (c,) the general solution is not defined at \(x=0\). Moreover, when \(x